Ecological Archives **A019-032-A1**

Melissa L. Snover and Selina S. Heppell. 2009. Application of diffusion approximation for risk assessments of sea turtle populations. *Ecological Applications* 19:774–785.

Appendix A. Details of the diffusion approximation method.

Methods for estimating parameters for diffusion approximation are reported in Dennis et al. (1991), Holmes (2001), Holmes and Fagan (2002), and Morris and Doak (2002). These methods are based on a model for exponential population growth in a randomly varying environment (Morris and Doak 2002)

$$N_{t+1} = N_t \lambda_t \qquad \text{(A.1)}$$

where N is the population size, t is time and λ_t is the population growth rate in year t. Diffusion approximation models estimate two key parameters used to make inferences regarding total population growth rates and quasi-extinction risks. These are $\hat{\mu}$, the arithmetic mean of the log population growth rate, and $\hat{\sigma}^2$, which accounts for sources of variability, including environmental and demographic stochasticity and observation error (Dennis et al. 1991, Morris and Doak 2002). Holmes (2001) introduced new methods for the estimation of $\hat{\sigma}^2$ which decomposes the variance into process and non-process error. Here we apply both methods to calculate $\hat{\sigma}^2$ and process error, $\hat{\sigma}_P^2$, on simulated time series to test if one method performs better than the other for sea turtle census data.

Running sums were calculated as follows

$$R_j = N_j + N_{j+1} \ldots + N_{j+(r-1)} \qquad \text{(A.2)}$$

where j = 1, 2, 3…(q-r+1), q is the number of censuses in the data set, r is the length of the running sum (2, 3, or 4) and R is used to represent population size at time j based on the running sums. For the methods presented by Dennis et al. (1991), the estimation of $\hat{\mu}$ and $\hat{\sigma}^2$ from annual census data, or running sums of census data, is straightforward from Eq. A.1 and is the mean and sample variance of log (R_{j+1}/R_j). Alternatively, these values can be found from a linear regression of $\sqrt{t_{j+1} - t_j}$ against $\log(R_{j+1} / Rj) / \sqrt{t_{j+1} - t_j}$ (Dennis et al. 1991, Morris and Doak 2002). The slope of the regression is $\hat{\mu}$ and 100(1-α) % confidence intervals can be approximated by a two-tailed Student's t distribution using the standard error of the slope. The residual mean square error of the regression is the estimate of $\hat{\sigma}^2$ and confidence intervals are constructed from a chi-square distribution with q–r degrees of freedom (Morris and Doak 2002).

Holmes (2001) approaches time-series analysis as an estimation problem with a hidden state, the true population size. Stating the problem in this way means that the data are modeling using the state-space model in Eq. 1 of Holmes and Fagan (2002). There are multiple estimation methods for state-space models of population data, including the slope estimation method (Holmes 2001, 2004), Kalman filter approaches (Lindley 2003), and restricted maximum likelihood (Staples et al 2004). In this paper we use the slope method which is both easy to implement and is robust to high ratios of non-process to process error (Holmes and Fagan 2002, Holmes 2004, Holmes et al. 2007). To do this, the log of the population growth rate is computed over different time intervals, τ =1, 2, …

$\tau_{\max}$ as $\log(R_{j+\tau}/ R_j)$. For each value of τ, the mean and variance of the log of the population growth rates is calculated and the slopes of regressions of those mean and variance values against τ give the estimates of $\hat{\mu}$ and $\hat{\sigma}_P^2$, process error, respectively. For these analyses we used $\tau = 1, 2, 3$. Methods for the construction of 95% confidence intervals around these values are given in Holmes and Fagan (2002).

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