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APPENDIX A. Description of the parameter estimation method.

Data:

The data set used to fit our model consisted of time series (1988–2000) measurements of diameter (mm) of trees in La Selva Biological Station, Costa Rica. Details of how these trees were chosen and measured are given in Clark and Clark (2006). We focus on the species *Simarouba amara* because this is a commercial timber species widely distributed in the tropics; thus our forward simulations can be viewed as timber yield projections. Point of diameter measurement had to be modified for some of the trees due to the growth of buttresses or other stem irregularities. Also, some trees had fewer than two diameter increment measurements, thus precluding the simultaneous estimation of process error and random individual effects. Thus, trees with less than two diameter increment measurements and trees with modified point of diameter measurement were discarded from our analysis.

State space model:

Here we describe the distributional assumptions for the state space model with random individual effects. This statistical model estimated the parameters used for all simulation models in the main text. Let $D_{i,t}$ and $\mathrm{D}_{\mathrm{i,t}}^{(\mathrm{o})}$ be the true and observed diameter (mm) of tree *i* in year *t*, respectively. We allow for measurement error by assuming that $\mathrm{N}\left(\mathrm{D}_{\mathrm{i,t}}^{(\mathrm{o})} \middle| \mathrm{D}_{\mathrm{i,t}}, \sigma_{\mathrm{obs}}^2\right)$. Furthermore, we assumed that true diameter increment is a function of a constant β_o and individual tree random effects. More explicitly, let $ln\left(D_{i,t+1} - D_{i,t}\right) = \beta_o + \varepsilon_i + \varepsilon_{i,t}$, where ε_i is the individual

tree i random effect and $\varepsilon_{i,t}$ is the process error. We assumed that $\varepsilon_i \sim N(0, \sigma^2_{\text{ind.eff}})$ and $\varepsilon_{i,t} \sim N(0, \sigma^2_{\text{proc}})$.

The estimation of the parameters $\beta_o, \varepsilon_i, \sigma^2_{\text{ind.eff}}, \sigma^2_{\text{obs}}$ and σ^2_{proc} was done using a Bayesian state space model (similar to the one described in Clark et al. (2007)). Priors for these parameters (except for ε_i) were $\text{N}(\beta_o | \mu_{\beta_o}, \sigma^2_{\beta_o})$, $\text{IG}(\sigma^2_{\text{ind.eff}} | s_{1,\text{ind.eff}}, s_{2,\text{ind.eff}})$, $\text{IG}(\sigma^2_{\text{obs}} | s_{1,\text{obs}}, s_{2,\text{obs}})$, $\text{IG}(\sigma^2_{\text{proc}} | s_{1,\text{proc}}, s_{2,\text{proc}})$, respectively. Hereafter, IG stands for the inverse-gamma distribution. A Gibbs sampler was designed using the conditional posteriors described in the appendix section "Conditional posteriors". In total, 500,000 iterations were performed, discarding the first half as burn-in. Convergence was confirmed by running several chains with over-dispersed starting points.

The comparison of the posterior and the prior distributions reveal that our prior mean for the measurement error variance σ^2_{obs} is close to the posterior mean, although it is not contained in the 95% credible interval (Fig. A1). Measurement error variance σ^2_{obs} was the smallest estimated variance and random individual tree effects accounted for more variability than the process variance.

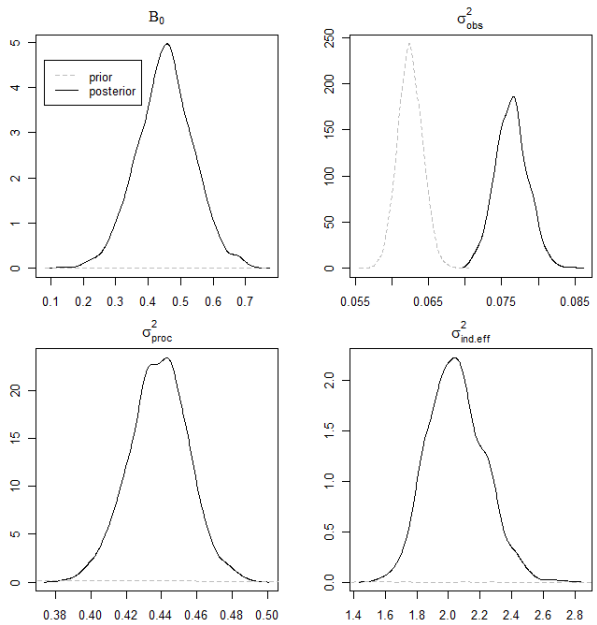

FIG. A1. Posterior (black lines) and prior (gray lines) parameter distributions from the Gibbs sampler.

Ten trees were randomly selected to be displayed, showing a close agreement between the posterior diameter distribution and the observed diameters (Fig. A2).

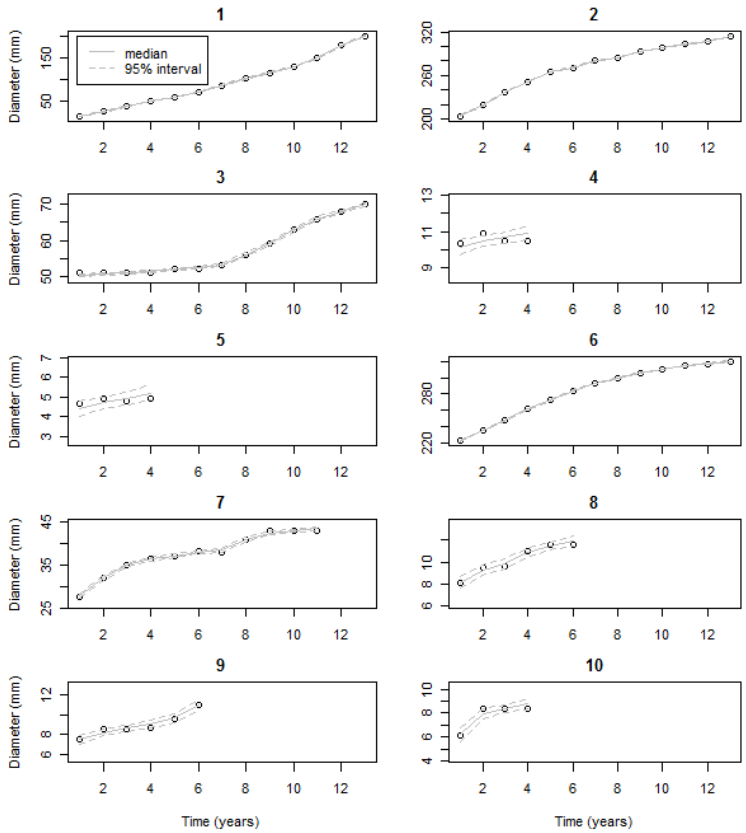

FIG. A2. Posterior diameter distribution (gray lines) vs. observed diameters (black circles) for ten randomly selected trees. Median and 95% credible intervals of the posterior distribution are shown with solid and dashed grey lines, respectively.

Conditional posteriors:

Here we describe the conditional posteriors for the state space model. A Gibbs sampler was used, which means that the random variables were sampled sequentially except when conditionally independent. The posterior distribution for some variables could be derived in closed form and thus could be sampled directly; other variables had to be sampled indirectly using a Metropolis within Gibbs step. All priors are vague except for the priors of the measurement error variance σ^2_{obs}.

- *For the states (sampled indirectly with a Metropolis step):*

  a) $\mathrm{D_{i,t}}$ *(intermediate states):*

$$\mathrm{p}\left(\mathrm{D_{i,t}}\right)$$

$$\propto \mathrm{N}\left(\ln\left(\mathrm{D_{i,t}} - \mathrm{D_{i,t-1}}\right) \middle| \beta_o + \varepsilon_i, \sigma^2_{\mathrm{proc}}\right)\mathrm{N}\left(\ln\left(\mathrm{D_{i,t+1}} - \mathrm{D_{i,t}}\right) \middle| \beta_o + \varepsilon_i, \sigma^2_{\mathrm{proc}}\right)\mathrm{N}\left(\mathrm{D^{(o)}_{i,t}} \middle| \mathrm{D_{i,t}}, \sigma^2_{\mathrm{obs}}\right)$$

  b) $\mathrm{D_{i,1}}$ *(first state):*

$$\mathrm{p}\left(\mathrm{D_{i,1}}\right) \propto \mathrm{N}\left(\ln\left(\mathrm{D_{i,2}} - \mathrm{D_{i,1}}\right) \middle| \beta_o + \varepsilon_i, \sigma^2_{\mathrm{proc}}\right)\mathrm{N}\left(\mathrm{D^{(o)}_{i,1}} \middle| \mathrm{D_{i,1}}, \sigma^2_{\mathrm{obs}}\right)$$

  c) $\mathrm{D_{i,T}}$ *(last state):*

$$\mathrm{p}\left(\mathrm{D_{i,T}}\right) \propto \mathrm{N}\left(\ln\left(\mathrm{D_{i,T}} - \mathrm{D_{i,T-1}}\right) | \beta_o + \varepsilon_i, \sigma^2_{\mathrm{proc}}\right)\mathrm{N}\left(\mathrm{D^{(o)}_{i,T}} | \mathrm{D_{i,T}}, \sigma^2_{\mathrm{obs}}\right)$$

- *For* σ^2_{proc} *(sampled directly)*:

$$\sigma^2_{\mathrm{proc}} \sim \mathrm{IG}\left(\mathrm{s_{1,proc}} + \frac{n(T-1)}{2}, \mathrm{s_{2,proc}} + \frac{\sum_{t=2}^{T}\sum_{i=1}^{n}\left(\ln\left(D_{i,t} - D_{i,t-1}\right) - (\beta_o + \varepsilon_i)\right)^2}{2}\right)$$

  The prior parameters $\mathrm{s_{1,proc}}$ and $\mathrm{s_{2,proc}}$ were set to 2.001 and 0.1001, respectively.

- *For* σ^2_{obs} *(sampled directly)*:

$$\sigma^2_{\mathrm{obs}} \sim \mathrm{IG}\left(\mathrm{s_{1,obs}} + \frac{nT}{2}, \mathrm{s_{2,obs}} + \frac{\sum_{t=1}^{T}\sum_{i=1}^{n}\left(D^{(o)}_{i,t} - D_{i,t}\right)^2}{2}\right)$$

  An earlier study for the same site suggested a maximum measurement error in diameter of 1 mm (Clark and Clark 1992). Assuming that 1 mm is equal to four standard deviations, this suggests a prior mean for the measurement error variance of 0.0625. Therefore, prior parameters were chosen to have this mean and to have the same weight as the data. Thus, $\mathrm{s_{1,obs}} = \frac{\mathrm{N_D}}{2}$ and $\mathrm{s_{2,obs}} = 0.0625 \times \left(\mathrm{s_{1,obs}} - 1\right)$, where $\mathrm{N_D}$ is the number of diameter observations.

- *For* β_o *(sampled directly)*:

$$\beta_o \sim N(Vv, V)$$

where $V^{-1} = \left(\frac{1}{\sigma^2_{\beta_o}} + \frac{(T-1)n}{\sigma^2_{proc}}\right)$ and $v = \left(\frac{\mu_{\beta_o}}{\sigma^2_{\beta_o}} + \frac{\sum_{t=2}^{T}\sum_{i=1}^{n}(\ln(D_{i,t}-D_{i,t-1})-\varepsilon_i)}{\sigma^2_{proc}}\right)$

The prior parameters μ_{β_o} and $\sigma^2_{\beta_o}$ were set to 1 and 1000, respectively.

- *For* ε_i *(sampled directly)*:

$$\varepsilon_i \sim N(Vv, V)$$

where $V^{-1} = \left(\frac{1}{\sigma^2_{ind.eff}} + \frac{(T-1)}{\sigma^2_{proc}}\right)$ and $v = \left(\frac{\sum_{t=2}^{T}(ln(D_{i,t}-D_{i,t-1})-\beta_o)}{\sigma^2_{proc}}\right)$

- *For* $\sigma^2_{ind.eff}$ *(sampled directly)*:

$$\sigma^2_{ind.eff} \sim IG\left(s_{1,ind.eff} + \frac{n}{2}, s_{2,ind.eff} + \frac{1}{2}\sum_{i=1}^{N}\varepsilon_i^2\right)$$

The prior parameters $s_{1,ind.eff}$ and $s_{2,ind.eff}$ were set to 2.001 and 0.1001, respectively. All the inverse-gamma distributions were parameterized as $IG(y|\alpha,\beta) = \frac{y^{-\alpha-1}e^{-\beta/y}}{(1/\beta)^{\alpha}\Gamma(\alpha)}$.

LITERATURE CITED

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