

APPENDIX B

We consider the case in which the dynamic spatial covariance is close to the following:

$$\begin{aligned} \hat{C}(d) &= V_{by} && \text{for } d > d_c \\ V_{by} \leq \hat{C}(d) &\leq V_{by} + V_{wy} && \text{for } 0 \leq d \leq d_c \end{aligned} \quad (\text{B.1})$$

Let Q_1 and Q_2 be two square regions, each having size $L \times L$. Suppose that they are separated by distance D , which is much larger than L ($L \ll D$). The average seed crop in area Q_1 and in area Q_2 are:

$$P_1(t) = \frac{1}{L^2} \sum_{\mathbf{x} \in Q_1} \varphi_{\mathbf{x}}(t) \quad \text{and} \quad P_2(t) = \frac{1}{L^2} \sum_{\mathbf{x} \in Q_2} \varphi_{\mathbf{x}}(t), \quad (\text{B.2})$$

respectively. We abbreviated the dependence of t in (B.2) for simplicity. Then the covariance between P_1 and P_2 is

$$\begin{aligned} \text{Cov}_t(P_1, P_2) &= \frac{1}{T} \sum_{t=1}^T (P_1(t) - \hat{P}_1)(P_2(t) - \hat{P}_2) \\ &= \frac{1}{T} \sum_{t=1}^T \frac{1}{L^2} \sum_{\mathbf{x} \in Q_1} \frac{1}{L^2} \sum_{\mathbf{y} \in Q_2} (\varphi_{\mathbf{x}}(t) - \hat{\varphi}_{\mathbf{x}})(\varphi_{\mathbf{y}}(t) - \hat{\varphi}_{\mathbf{y}}) \\ &= \frac{1}{L^4} \sum_{\mathbf{x} \in Q_1} \sum_{\mathbf{y} \in Q_2} C(\mathbf{x}, \mathbf{y}) \end{aligned} \quad (\text{B.3})$$

from Eq. (B.2) and Eq. (8a). Since our model generates spatial patterns that are isotopic, the crosscovariance between two time series sampled from \mathbf{x} and \mathbf{y} is a function of their

distance, and hence $C(\mathbf{x}, \mathbf{y}) \approx \hat{C}(\|\mathbf{x} - \mathbf{y}\|)$. Using this expression and the assumption of Eq. (B.1), Eq. (B.3) beomes,

$$\text{Cov}_t(P_1, P_2) = V_{by}. \quad (\text{B.4})$$

In a similar way, we have

$$\begin{aligned} \text{Var}_t(P_1(t)) &= \text{Var}_t(P_2(t)) \\ &= \frac{1}{L^4} \sum_{\mathbf{x} \in Q_1} \sum_{\mathbf{y} \in Q_1} \hat{C}(\|\mathbf{x} - \mathbf{y}\|). \end{aligned} \quad (\text{B.5})$$

Using Eq. (B.1), it has an upper bound:

$$\begin{aligned} \text{Var}_t(P_1(t)) &\leq \frac{1}{L^4} \sum_{\substack{\mathbf{x} \in Q_1 \\ \mathbf{y} \in Q_1 \\ \|\mathbf{x} - \mathbf{y}\| > d_c}} V_{by} + \frac{1}{L^4} \sum_{\substack{\mathbf{x} \in Q_1 \\ \mathbf{y} \in Q_1 \\ \|\mathbf{x} - \mathbf{y}\| \leq d_c}} (V_{by} + V_{wy}) \\ &\approx V_{by} + \frac{\pi d_c^2}{L^2} V_{wy} \end{aligned} \quad (\text{B.6})$$

The correlation coefficient between P_1 and P_2 is

$$\frac{\text{Cov}_t(P_1, P_2)}{\text{Var}_t(P_1(t))} \geq \frac{V_{by}}{V_{by} + \frac{\pi d_c^2}{L^2} V_{wy}}, \quad (\text{B.7})$$

which can be very close to 1 when the size of the sampled area L is much larger than the correlation distance d_c ($L \gg d_c$). This argument holds if V_{by} is positive, however small it may be, and hence whenever there is an element of global coupling over the forest.