Effects of Successional Dynamics on Metapopulation Persistence: Appendix

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Appendix A. Technical details of the models

Colonization model

The colonization rate model can be derived from the following assumptions. It's an entirely standard stochastic colonization model, so if the description in the text made sense to you, it's safe to skip this section. Otherwise, an introductory text on stochastic processes (e.g. Grimmett and Stirzaker 1992, section 6.8 and associated homework exercises) can be consulted for background information on Poisson processes.

We assume that the times at which an Occupied patch emits colonists is given by a Poisson point process on the line with constant intensity Q, resulting from a constant number of individuals in the patch with a constant propensity to migrate. Because our model classifies patches by successional state without tracking population dynamics, the value of Q is assumed to be the same for all Occupied patches and constant over time. Thus, when there are m occupied patches, the departure times of migrants from all patches combined is a Poisson point process with intensity Qm.

In the nonspatial model, emigrants are dispersed uniformly at random over the entire territory occupied by the population. Each emigrant has some probability a of entering any given patch (hence probability Na of entering some patch, where N is the total number of patches). Note that emigrants do not select among patches: any patch, whether Vacant, Occupied, or Refractory, is equally likely to be the recipient. Thus when m patches are Occupied the number of migrants arriving into a given Vacant patch is a Poisson point process with intensity $aQm = cm$ with $c=aQ$, which has the properties stated in the text.

In the spatial model, emigrants disperse non-uniformly with a higher probability of entering nearby patches. Consider an Occupied patch *j* emitting emigrants as a Poisson process with rate *Q*, and a patch *i* (in any successional stage) at distance d_{ij}. The probability that an emigrant from patch j enters patch *i* is assumed to be $C_1 \exp(-\alpha d_{ij}) + C_2$, where C_1, C_2, α are such that the probability of an emigrant landing in some patch is at most one: $C_1 \sum_{j \neq i} \exp(-\alpha d_{ij}) + C_2 \leq 1$ for all *i*, with the sum running over all other patches in the network). Arrivals into a Vacant patch *i* thus constitute a Poisson point process with rate $c_1 \sum_j \exp(-\alpha d_{ij}) 1_{\{\text{Patch j is Occupied}\}} + c_2$ where $c_i = QC_i$, corresponding to the colonization rate stated in the main text.

Large-network deterministic approximation

The large-network deterministic approximation involves two assumptions. The first is to consider an effectively infinite network, whose state is specified by $n(a,t)$, the frequency distribution (density) of patch ages *a* at time *t* (the age of a patch is the time since it was colonized). The second is to assume that stage transitions (e.g., the number of vacant patches being colonized per unit time) occur at exactly their expected rates, given the current number of patches in each stage. Consider first the case of constant refractory stage duration *R*. Under the stated assumptions, the age-distribution of Occupied and Refractory patches ($0 < a < 1 + R$) follow the McKendrick-VonFoerster equation without mortality:

$$\partial n / \partial t + \partial n / \partial a = 0 . \tag{1}$$

Occupied patches are "born" as a result of colonization:

$$n(0,t) = cO(t)V(t) \tag{2}$$

where $O(t) = \int_0^1 n(a,t)da$ is the total number of occupied patches. Vacant patches are added by patches increasing in age past 1+ R, and lost through colonization,

$$dV/dt = n(1+R,t) - cO(t)V(t)\,. \tag{3}$$

Equations (1)-(3) can be solved numerically by considering discrete age-classes and time-steps of duration $dt \ll 1$, and iterating the age-class transitions described by (1) and the inputs and losses specified by (2) and (3). Note that equation (3) is actually redundant, since the total number of patches N is constant, hence $V = N - \int_0^{1+R} n(a,t)da$.

If the duration of the refractory period is random with exponential distribution and mean E[R], then equations (1) and (2) still apply for Occupied patches (0<a<1), and there are ordinary differential equations for both the Refractory and Vacant patches:

$$\begin{aligned} dR/dt &= n(1,t) - R(t)/E[R] \\ dV/dt &= R(t)/E[R] - cO(t)V(t) \end{aligned} \quad . \tag{4}$$

If the Refractory period is random but non-exponentially distributed, the large-network deterministic approximation is given by (1) with a "mortality" term on the right-hand side for an age-dependent "death" rate of Refractory (a>1) patches into the Vacant state. The total "death" rate then replaces $n(1+R,t)$ as the input rate for $V(t)$ in equation (3).

References

Grimmett, G. R. and D. R. Stirzaker. 1992. Probability and Random Processes, 2nd edition. Oxford University Press, Oxford UK.