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APPENDIX A: The MaxEnt Machinery

We seek the least biased estimate of the functional form of a probability distribution $p(n)$ that is subject to a set of K constraints that are accepted from prior knowledge and that can be expressed in the form of K equations:

$$\sum_n f_k(n) p(n) = \langle f_k \rangle \quad (\text{A.1})$$

where n is summed over all of its possible values, $\langle f_k \rangle$ is the numerical value of the average of f_k , and the index k runs from 1 to K . It was shown by Jaynes (9) that the best inference as to the shape of $p(n)$ is the function that maximizes the “information entropy”

$$I = -\sum_n p(n) \ln(p(n)) \quad (\text{A.2})$$

subject to those constraints. Maximization is carried out using the method of Lagrange multipliers. We are assuming a uniform reference entropy here (sensu Jaynes) because we can construct the continuous formulation of the problem as the limiting case of a discrete formulation and because there is no a priori reason to choose anything other than a uniform reference. The maximization procedure yields:

$$p(n) = \frac{e^{-\sum_{k=1}^K \lambda_k f_k(n)}}{Z(\lambda_1, \lambda_2, \dots, \lambda_K)} \quad (\text{A.3})$$

where Z , the partition function, is given by:

$$Z(\lambda_1, \lambda_2, \dots, \lambda_K) = \sum_n e^{-\sum_{k=1}^K \lambda_k f_k(n)} \quad (\text{A.4})$$

and the λ_k are given by the solutions to:

$$\frac{\partial \ln(Z)}{\partial \lambda_k} = -\langle f_k \rangle \quad (\text{A.5})$$

A practical example of the use of this equation is in Appendix B. Eqs. A.1 to A.5 readily generalize to joint probability distributions, such as $R(n, \varepsilon)$ in the text.

