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## Appendix E: Stability analysis when risk is inverse gaussian distributed

In the case when the distribution of risk is an inverse gaussian distribution with mean c and coefficient of variation $CV$, we have (Chhikara and Folks 1989)

$$f(P_t) = \int_{x=0}^{\infty} p(x)\exp(-xP_t)dx = \exp\left(\frac{1}{CV^2}\left(1-\sqrt{1+2cCV^2P_t}\right)\right). \tag{E.1}$$

Thus the discrete-time model

$$H_{t+1} = RH_t \int_{x=0}^{\infty} p(x)\exp(-xP_t)dx \tag{E.2}$$

$$P_{t+1} = k(RH_t - H_{t+1}) \tag{E.3}$$

becomes

$$H_{t+1} = RH_t \exp\left(\frac{1}{CV^2}\left(1-\sqrt{1+2cCV^2P_t}\right)\right) \tag{E.4}$$

$$P_{t+1} = k(RH_t - H_{t+1}). \tag{E.5}$$

The equilibrium of this model is given as[1]

$$H^* = \frac{.5CV^2(\log(R))^2+\log(R)}{kc(R-1)},\;\; P^* = \frac{.5CV^2(\log(R))^2+\log(R)}{c}. \tag{E.6}$$

Using the stability condition (equation (B.22) in Appendix B) we have that the above equilibrium is stable, if and only if,

$$\frac{.5CV^2(\log(R))^2+\log(R)}{CV^2\log(R)+1} < \frac{R-1}{R}. \tag{E.7}$$

With $CV = 1.31$ for the distribution of risk in Figure 2, we calculate from the above inequality that the host-parasitoid equilibrium is stable for $1 < R < 2$ and unstable for $R > 2$. One can also see that if $R$ is chosen larger than 4.92 then inequality (E.7) can never hold for any value of $CV^2$. Hence in this case even infinite amount of $CV$ will not stabilize the equilibrium.

[1] $\log(x)$ denotes the natural logarithm of $x$.

## REFERENCES

Chhikara, R. S., and J. L. Folks, 1989. The inverse gaussian distribution: theory, methodology, and applications. Marcel Dekker, New York.