Ecological Archives E093-220-A7

Roland Langrock, Ruth King, Jason Matthiopoulos, Len Thomas, Daniel Fortin, and Juan M. Morales. 2012. Flexible and practical modeling of animal telemetry data: hidden Markov models and extensions. *Ecology* 93:2336–2342. http://dx.doi.org/10.1890/11-2241.1

Appendix G. Verification of the HMM representation being an approximate representation of the example HSMM given in Section 4.1.

We show that

$$p_1^*(r) = p_1(r), \qquad \text{for } r \leq N^*,$$

where $p_1^*(r)$ denotes the p.m.f. of the dwell-time distribution in state aggregate I of the approximating HMM, and $p_1(r)$ denotes the p.m.f. of the dwell-time distribution in state 1 of the HSMM that is to be approximated. First of all, we note that a stay in state aggregate I begins in state 1 (follows from the definitions of the HMM state transition probabilities in the $(N^* + 1)$th row of Γ and of the initial state distribution). Consequently,

$$p_1^*(1) = \gamma_{1,N^*+1} = c(1) = p_1(1)$$

$$p_1^*(2) = \gamma_{12} \cdot \gamma_{2,N^*+1} = \big(1 - c(1)\big) \cdot c(2) = \big(1 - p_1(1)\big) \cdot \frac{p_1(2)}{1 - p_1(1)} = p_1(2)$$

$$\begin{aligned} p_1^*(3) = \gamma_{12} \cdot \gamma_{23} \cdot \gamma_{3,N^*+1} &= \big(1 - c(1)\big) \cdot \big(1 - c(2)\big) \cdot c(3) \\ &= \big(1 - p_1(1)\big) \cdot \frac{1 - p_1(1) - p_1(2)}{1 - p_1(1)} \cdot \frac{p_1(3)}{1 - p_1(1) - p_1(2)} = p_1(3) \end{aligned}$$

$$\vdots$$

$$\begin{aligned} p_1^*(N^*) = \gamma_{12} \cdot \gamma_{23} \cdot \ldots \cdot \gamma_{N^*-1,N^*} \cdot \gamma_{N^*,N^*+1} & \\ = \big(1 - c(1)\big) \cdot \big(1 - c(2)\big) \cdot \ldots \cdot \big(1 - c(N^* - 1)\big) \cdot c(N^*) & \\ = \big(1 - p_1(1)\big) \frac{1 - p_1(1) - p_1(2)}{1 - p_1(1)} \cdot \ldots & \\ \cdot \frac{1 - p_1(1) - p_1(2) - \ \ldots - p_1(N^* - 1)}{1 - p_1(1) - \ \ldots - p_1(N^* - 2)} & \\ \cdot \frac{p_1(N^*)}{1 - p_1(1) - p_1(2) - \ \ldots - p_1(N^* - 1)} = p_1(N^*) & \end{aligned}$$

Alternatively, the statement can be proven via induction (Langrock and Zucchini 2011).

Literature Cited

Langrock, R., and W. Zucchini. 2011. Hidden Markov models with arbitrary dwell-time distributions. Computational Statistics and Data Analysis 55:715–724.