

McClintock, B. T., D. J. F. Russell, J. Matthiopoulos, and R. King. 2013. Combining individual animal movement and ancillary biotelemetry data to investigate population-level activity budgets. *Ecology*.

## APPENDIX A

### *Prior specifications, prior sensitivity, and goodness of fit for the harbor seal example*

*Prior specification.* For reference, we first review the model specification. We have assumed

$$\left[ s_{n,t} \mid \mathbf{a}_n, \mathbf{b}_n, z_{n,t} = i \right] \sim \text{Weibull}(a_{n,i}, b_{n,i})$$

for state-specific scale parameter  $a_{n,i} > 0$  and shape parameter  $b_{n,i} > 0$ , where

$$\log(a_{n,i}) \sim N(\alpha_{i,j}, \sigma_{\alpha_{i,j}}^2)$$

and

$$\log(b_{n,i}) \sim N(\beta_{i,j}, \sigma_{\beta_{i,j}}^2)$$

for  $i = R, F, T$  and  $n = 1, \dots, N$ , where  $j = 1$  if individual  $n$  is male and  $j = 2$  otherwise. We also assume

$$\left[ \phi_{n,t} \mid \boldsymbol{\rho}_n, z_{n,t} = i \right] \sim \text{wCauchy}(\phi_{n,t-1}, \rho_{n,i})$$

with bearing  $0 \leq \phi_{n,t} < 2\pi$  and state-specific directional persistence  $0 \leq \rho_{n,i} < 1$ , where

$$\tanh^{-1}(\rho_{n,i}) \sim N(r_{i,j}, \sigma_{r_{i,j}}^2).$$

For the proportion of each time step spent diving below 1.5m ( $\omega_{n,t}$ ), we assume

$$\left[ \omega_{n,t} \mid \boldsymbol{\nu}, \boldsymbol{\delta}, z_{n,t} = i \right] \sim \text{Beta}(\nu_{i,j}, \delta_{i,j})$$

where  $v_{F,j} = v_{T,j}$  and  $\delta_{F,j} = \delta_{T,j}$ .

In terms of trajectory, we would expect speeds to be low during resting and low to moderate during foraging, with little directional persistence. During transit, we would expect high speeds and high directional persistence. To reflect these expectations, we assigned priors for the movement process parameters as follows. We assigned uniform priors on the inverse log scale for the population-level step length parameters

$$\begin{aligned}\exp(\alpha_{R,j}) &\sim \text{Unif}(0, \exp(\alpha_{F,j})) \\ \exp(\alpha_{F,j}) &\sim \text{Unif}(\exp(\alpha_{R,j}), \exp(\alpha_{T,j})) \\ \exp(\alpha_{T,j}) &\sim \text{Unif}(\exp(\alpha_{F,j}), s_{\max}) \\ \exp(\beta_{i,j}) &\sim \text{Unif}(0, 5)\end{aligned}$$

and constrained the individual-level step length parameters on the real scale

$$\begin{aligned}0 &< a_{n,R} < a_{n,F} < a_{n,T} < s_{\max} \\ b_{n,i} &< 5\end{aligned}$$

for  $i = R, F, T$ ,  $n = 1, \dots, N$ , and  $j = 1, 2$  (male and female, respectively), where  $s_{\max}$  is the maximum possible distance an individual can travel within each time step. We specified  $s_{\max} = 14.4$  km based on a maximum sustainable speed of 2 m per second (Cunningham et al. 2009). We also constrained predicted locations  $(X_{n,t-1}, Y_{n,t-1})$  and  $(X_{n,t}, Y_{n,t})$  such that  $s_{n,t} \leq s_{\max}$ . For computational efficiency and simplicity, we did not prevent predicted locations from being inland. Although these instances were relatively rare, such constraints would facilitate more

realistic predictions about harbor seal movements in the vicinity of land. This is particularly true for time steps with no observed locations (i.e.,  $k_{n,t} = 0$ ).

We assigned uniform priors on the hyperbolic tangent of the population-level directional persistence parameters

$$\tanh(r_{R,j}) \sim \text{Unif}(0,1)$$

$$\tanh(r_{F,j}) \sim \text{Unif}(0, \tanh(r_{T,j}))$$

$$\tanh(r_{T,j}) \sim \text{Unif}(\rho_{\min}, 1)$$

and constrained the individual-level directional persistence parameters on the real scale

$$\rho_{n,F} < \rho_{n,T}$$

$$\rho_{\min} < \rho_{n,T}$$

where  $\rho_{\min}$  is the minimum mean vector length for the transit state. We specified  $\rho_{\min} = 0.75$  to inform a relatively strong directional persistence when in the transit state.

For the (state-dependent) Beta prior distributions for  $\omega_{n,t}$ , we assigned hyperpriors based on the expected relationships between time spent diving below 1.5m and the three movement behavior states. Relative to the foraging and transit states, we expected less time spent diving below 1.5m during the resting state. We also expected dive times to be similar for the foraging and transit states. We therefore assigned the uniform priors

$$\nu_{R,j} \sim \text{Unif}(1, \delta_{R,j})$$

$$\nu_{F,j} \sim \text{Unif}(\delta_{F,j}, 10)$$

$$\delta_{R,j} \sim \text{Unif}(\nu_{R,j}, 10)$$

$$\delta_{F,j} \sim \text{Unif}(1, \nu_{F,j})$$

where  $\nu_{T,j} = \nu_{F,j}$  and  $\delta_{T,j} = \delta_{F,j}$ . We note that under this prior specification,

$[\omega_{n,t} | \boldsymbol{\nu}, \boldsymbol{\delta}, z_{n,t} = i] \sim \text{Unif}(0,1)$  when  $\nu_{i,j} = \delta_{i,j} = 1$ . We set the lower bound at 1 to prohibit bimodal (i.e., "bathtub-shaped") distributions. We specified the upper bound at 10 to ensure that at the extremes [i.e., Beta(1,10) or Beta(10,1)] there would remain some prior density for all three states when  $\omega_{n,t} \approx 0.5$ , but little prior density for resting when  $\omega_{n,t} > 0.5$  or for foraging and transit when  $\omega_{n,t} < 0.5$  (Fig. A1). Additional prior specifications are reported in Table A1.

*Prior sensitivity.* We found some sensitivity to the hyperprior specification of the (state-dependent) Beta distributions for  $\omega_{n,t}$ . This is evident in the estimated posterior densities at the lower or upper boundaries for  $\delta_{R,1}$ ,  $\delta_{R,2}$ , and  $\delta_{F,1}$  (see *Appendix C*). Although we believe the posterior densities at the lower bound are biologically justified in prohibiting bimodal distributions, we suspect prior sensitivity at the upper bound may be related to a lack of model fit attributable to mid-interval state switches between resting and foraging or transit. For example, if an individual switched from resting to transit mid-interval, this time step could potentially be assigned to any of the three movement behavior states (depending on the average speed and direction of movement over the entire interval). While we expected  $\omega_{n,t} \ll 0.5$  for the resting state and  $\omega_{n,t} \gg 0.5$  for the foraging and transit states, about 10% of the diving proportions were between 0.4 and 0.6. We therefore conducted another analysis to investigate prior sensitivities to the upper bound using the following less-informative hyperpriors:

$$\nu_{R,j} \sim \text{Unif}(1, \delta_{R,j})$$

$$\nu_{F,j} \sim \text{Unif}(\delta_{F,j}, 100)$$

$$\delta_{R,j} \sim \text{Unif}(\nu_{R,j}, 100)$$

$$\delta_{F,j} \sim \text{Unif}(1, \nu_{F,j}),$$

where  $\nu_{T,j} = \nu_{F,j}$  and  $\delta_{T,j} = \delta_{F,j}$ .

With these hyperpriors, we found little difference for the females, but we did find differences for the males. Estimated male activity budgets using these less-informative hyperpriors were 0.53 (95%HPDI: 0.52–0.53) for resting, 0.38 (0.37–0.39) for foraging, and 0.09 (0.09–0.10) for transit. Movement characteristics were similar, but very few time steps with  $\omega_{n,t} < 0.6$  were assigned to foraging or transit. There were also many time steps with  $\omega_{n,t} > 0.5$  that were assigned to resting (Figs. A2 and A3). In short, allowing more peaked distributions for  $\omega_{n,t}$  produced what appear to be biologically unreasonable results for males. We therefore found the original hyperprior specification justifiable and more reasonable, but acknowledge that mid-interval state switches or some structural deficiencies may be inducing a lack of model fit. It is also possible that resting on the sea floor or display dives during the breeding season are more likely to be assigned to resting than to foraging with the less-informative prior specification.

We also investigated alternative priors for  $\omega_{n,t}$  using the logit-normal distribution. Specifically, we assigned  $[\omega_{n,t} | \boldsymbol{\mu}, \boldsymbol{\sigma}, z_{n,t} = i] \sim \text{Logit-Normal}(\mu_{i,j}, \sigma_{i,j}^2)$  for  $i = R, F, T$  and  $j = 1, 2$  where  $\mu_{F,j} = \mu_{T,j}$  and  $\sigma_{F,j}^2 = \sigma_{T,j}^2$ . Although the logit-normal distribution does not have an analytical mean or variance, we used numerical methods to assign hyperpriors that yield logit-normal prior distributions that closely resemble the original Beta priors (and still prohibit bimodal densities):

$$\mu_{R,j} \sim \text{Unif}(-3.258, 0)$$

$$\mu_{F,j} \sim \text{Unif}(0, 3.258)$$

$$\sigma_{R,j}^2 \sim \text{Unif}(0, 1.69^2)$$

$$\sigma_{F,j}^2 \sim \text{Unif}(0, 1.69^2)$$

With this prior specification, we achieved virtually identical results as with the original Beta priors. However, we found greater prior sensitivity when using the following less-informative hyperpriors for the logit-normal parameters:

$$\mu_{R,j} \sim \text{N}(-3.258, 1)$$

$$\mu_{F,j} \sim \text{N}(3.258, 1)$$

$$\sigma_{R,j}^2 \sim \text{Unif}(0, 1.69^2)$$

$$\sigma_{F,j}^2 \sim \text{Unif}(0, 1.69^2)$$

This specification resulted in biologically unreasonable results for both sexes, with only those time steps with  $\omega_{n,t} = 0.0$  assigned to the resting state (16% and 10% of time steps were assigned to resting, 71% and 81% to foraging, and 13% and 8% to transit for males and females, respectively).

*Goodness of fit.* We used residual plots and Bayesian posterior predictive checks (e.g., Gelman et al. 2005) to help assess goodness of fit. The estimated observation model error terms,  $\varepsilon_{x_{n,t,i}}$  and  $\varepsilon_{y_{n,t,i}}$ , relative to  $j_{n,t,i}$  (the timing of each observation within each 120min time step) provided no indication of lack of fit of our normally-distributed observation model when using both location and dive activity data (Fig. A4). However, normal QQ plots for  $\varepsilon_{x_{n,t,i}}$  and  $\varepsilon_{y_{n,t,i}}$  for

the  $N = 17$  seal tracks (Fig. A5) suggest shorter tails than would be expected from a normal distribution (for reference, Fig. 4a corresponds to individual  $n = 8$  and Fig. 4b corresponds to individual  $n = 1$ ). We believe this "short-tail" tendency beginning at 2–3 standard deviations is a result of step lengths ( $s_t$ ) being constrained by the maximum speed of 2 m/s, i.e.,  $\max(s_t) = 14.4$  km. In other words, this prior distribution on  $s_t$  reduces the potential for particularly large  $\varepsilon_{x_{n,t,i}}$  and  $\varepsilon_{y_{n,t,i}}$ . We note that increasing the maximum speed  $>2$  m/s is not biologically realistic for this species.

Auto-correlation plots for  $\varepsilon_{x_{n,t,i}}$  and  $\varepsilon_{y_{n,t,i}}$  (Fig. A6) indicate correlations drop off relatively rapidly for the individual tracks, although some individuals exhibit what appear to be somewhat unusual lower-order correlations. The more unusual lower-order correlation patterns are the price we pay for discretization of the movement path into 120min step lengths (e.g., imagine drawing a straight line through an S-shaped path, much like a dollar symbol \$). Because these correlations are a function of the observed location data and the time step length, we don't believe a reasonable a priori error structure could be devised to accommodate this. Given our goal was not to perfectly recreate each individual movement path (but only to reasonably characterize them into 120 min time steps), we do not believe much would be gained by attempting a more complicated error structure to accommodate these departures from normality.

As an additional assessment of goodness of fit, we adapted the approach proposed by Gelman et al. (2005) for Bayesian posterior predictive checks and  $p$  values with missing data and latent variables. The basic idea is to simulate the data  $(\mathbf{x}^{rep}, \mathbf{y}^{rep}, \boldsymbol{\omega}^{rep})$  from each draw of the posterior and compare the simulated data (from the model) to the observed data using a discrepancy function. Specifically, at each iteration  $g$  of the MCMC chain, we sample

$$\begin{aligned}x_{n,t,i}^{rep(g)} &\sim N\left(j_{n,t,i}X_{t-1}^{(g)} + (1-j_{n,t,i})X_t^{(g)}, \sigma_x^{2(g)}\right), \\y_{n,t,i}^{rep(g)} &\sim N\left(j_{n,t,i}Y_{t-1}^{(g)} + (1-j_{n,t,i})Y_t^{(g)}, \sigma_y^{2(g)}\right),\end{aligned}$$

and

$$\left[\omega_{n,t}^{rep(g)} \mid z_{n,t}^{(g)} = i\right] \sim Beta\left(\nu_{i,j}^{(g)}, \delta_{i,j}^{(g)}\right).$$

Following Gelman et al. (2005), our discrepancy functions for  $\mathbf{x}$  and  $\mathbf{y}$  were  $sd(\mathbf{x})$  and  $sd(\mathbf{y})$ , where  $sd$  is the standard deviation. The posterior predictive  $p$  values are then  $\Pr\left(sd(\mathbf{x}^{rep(g)}) > sd(\mathbf{x})\right)$  and  $\Pr\left(sd(\mathbf{y}^{rep(g)}) > sd(\mathbf{y})\right)$ . The resulting posterior predictive  $p$ -values were 0.54 and 0.53, respectively, indicating no discrepancy between our replicated data from the model and the observed data.

For the dive activity data, we used  $sd(\boldsymbol{\omega})$  and  $\sum_n \sum_t \boldsymbol{\omega}$  as discrepancy functions, the latter being a measure of the total amount of time the population spent diving below 1.5 m. The posterior predictive  $p$ -values are then  $\Pr\left(sd(\boldsymbol{\omega}^{rep(g)}) > sd(\boldsymbol{\omega})\right)$  and  $\Pr\left(\sum_n \sum_t \boldsymbol{\omega}^{rep(g)} > \sum_n \sum_t \boldsymbol{\omega}\right)$ .

We did find evidence of our replicated dive data being slightly underdispersed relative to the observed dive activity data, with  $sd(\boldsymbol{\omega}) = 0.33$  and mean  $sd(\boldsymbol{\omega}^{rep(g)}) = 0.32$  (estimated  $p$ -value = 0.0). We do not find this slight discrepancy to be particularly alarming (because the magnitude of the discrepancy is so small), but this nonetheless is an indication of some lack of fit (most likely in the extreme tails) of the estimated Beta distribution parameters given these data and prior specifications. The other discrepancy measure,  $\sum_n \sum_t \boldsymbol{\omega}$ , indicated no strong evidence of a discrepancy ( $p$  value = 0.17) between our replicated data from the model and the observed data.

These goodness of fit diagnostics for the observation model ( $\varepsilon_{x_{n,j,i}}$  and  $\varepsilon_{y_{n,j,i}}$  or  $x_{n,j,i}^{rep}$  and  $y_{n,j,i}^{rep}$ )



$y_{n,i,j}^{rep}$ ) were similar for the analysis utilizing location data only. These were therefore not particularly useful for comparing the two analyses. We instead used measures relative to the observed dive activity data for model comparisons (see *Example results and discussion*).

#### LITERATURE CITED

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Gelman, A., I. Van Mechelen, G. Verbeke, D. F. Heitjan, and M. Meulders. 2005. Multiple imputation for model checking: completed-data plots with missing and latent data. *Biometrics* 61:74–85.

TABLE A1. Additional prior specifications for a Bayesian analysis of harbor seal activity budgets utilizing both location and diving data. The movement behavior states include resting ( $R$ ), foraging ( $F$ ), and transit ( $T$ ).

Parameter	Description	Prior distribution
$\phi_{n,0}$	Direction (or bearing) of movement for initial time step $t = 0$ for individuals $n = 1, \dots, N$ .	$\text{Unif}(0, 2\pi)$
$\sigma_{\alpha_{z,j}}^2$	Log-scale variance for individual-level random effect on the step length scale parameter $(\alpha_{z,s})$ of males ( $j = 1$ ) and females ( $j = 2$ ) for states $z = R, T, F$ .	$\Gamma^{-1}(3, 2)$
$\sigma_{\beta_{z,j}}^2$	Log-scale variance for individual-level random effect on the step length shape parameter $(\beta_{z,j})$ of males ( $j = 1$ ) and females ( $j = 2$ ) for states $z = R, T, F$ .	$\Gamma^{-1}(3, 2)$
$\sigma_{\rho_{z,j}}^2$	Variance for individual-level random effect on the directional persistence parameter $(\rho_{z,j})$ of males ( $j = 1$ ) and females ( $j = 2$ ) for states $z = R, T, F$ .	$\Gamma^{-1}(3, 2)$
$\sigma_x^2$	Measurement error variance for longitudinal coordinates of observed locations $(x_{n,t,i}, y_{n,t,i})$ .	$\Gamma^{-1}(1, 10^{-6})$
$\sigma_y^2$	Measurement error variance for latitudinal coordinates of observed locations $(x_{n,t,i}, y_{n,t,i})$ .	$\Gamma^{-1}(1, 10^{-6})$
$\psi_{k,j}$	The $k$ th row vector of the state transition probability matrix for males ( $j = 1$ ) and females ( $j = 2$ ), with each	$\text{Dirichlet}(1, 1, 1)$

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element  $(\psi_{k,i,j})$  corresponding to the switching  
probability from state  $k$  at time  $t - 1$  to state  $i = R, T, F$   
at time  $t$ .

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FIG. A1. State-dependent prior densities for the proportion of each time step spent below 1.5m  
( $\omega$ ) at the extremes of the hyperprior distributions. These state-dependent prior distributions are

$[\omega | z = R] \sim \text{Beta}(1, 10)$  for the resting state and  $[\omega | z = F, T] \sim \text{Beta}(10, 1)$  for the foraging and transit states.

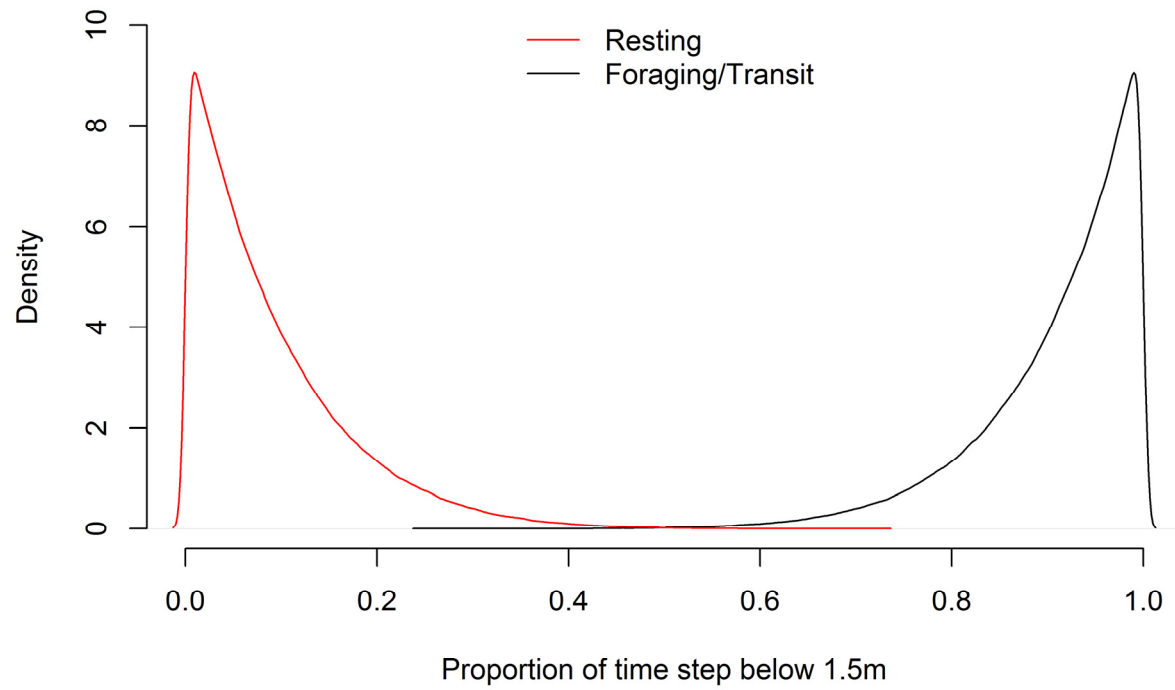


FIG. A2. Posterior histogram of sex- and state-dependent proportions of time step spent diving below 1.5m ( $\omega_{n,t}$ ) for 17 harbor seals in the UK when using a less-informative hyperprior specification of the Beta distribution for  $\omega_{n,t}$  (shown in red).

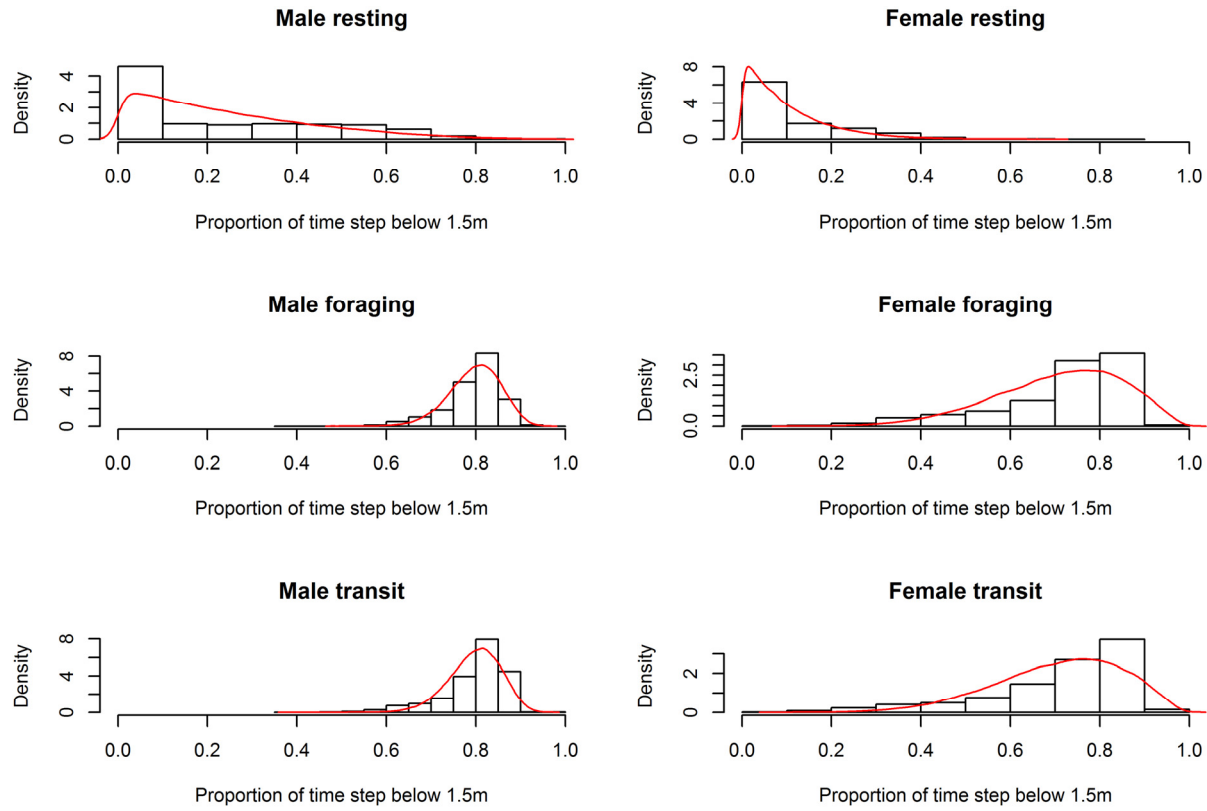


FIG. A3. Estimated bivariate densities of harbor seal step length and proportion of time step spent diving below 1.5m ( $\omega_{n,t}$ ) when using a less-informative hyperprior specification of the Beta distribution for  $\omega_{n,t}$ . Separate densities were estimated for males and females with three distinct movement behavior states ("resting", "foraging", and "transit"), where darker shades indicate higher relative densities.

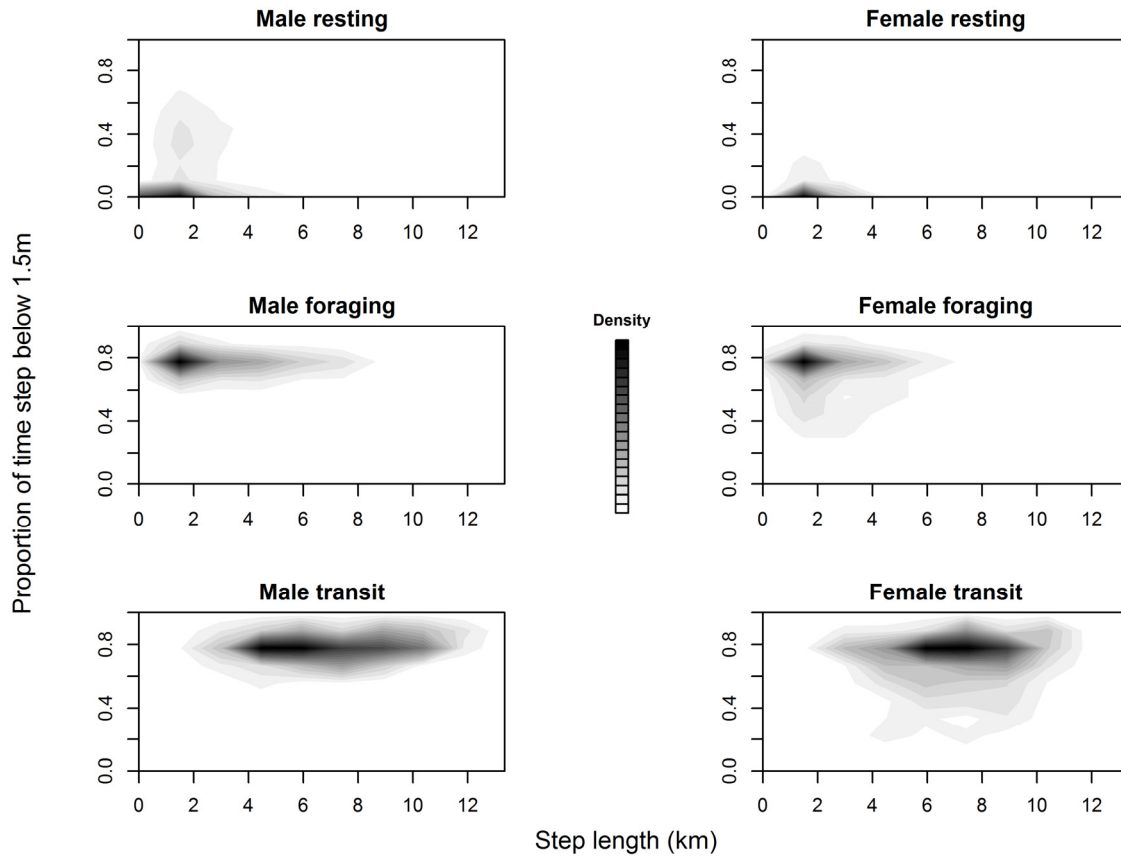


FIG. A4. Posterior means of the observation model error terms,  $\varepsilon_{x_{n,t,i}}$  and  $\varepsilon_{y_{n,t,i}}$ , relative to  $j_{n,t,i}$  (the timing of each observation within each 120-min time step).

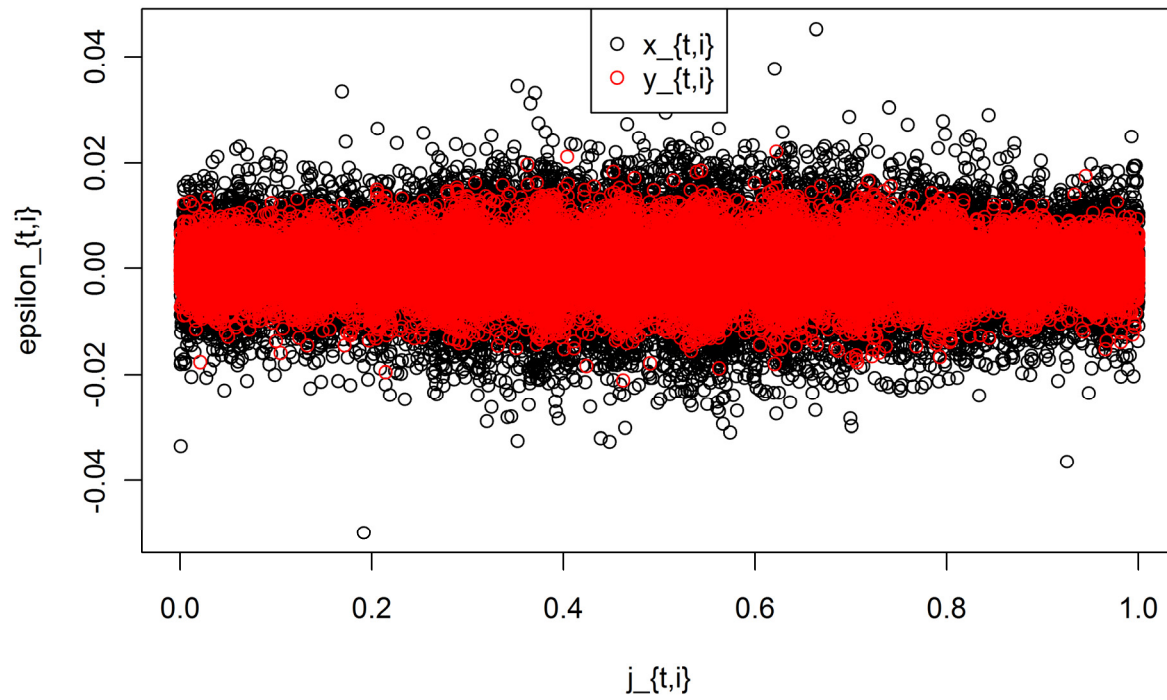
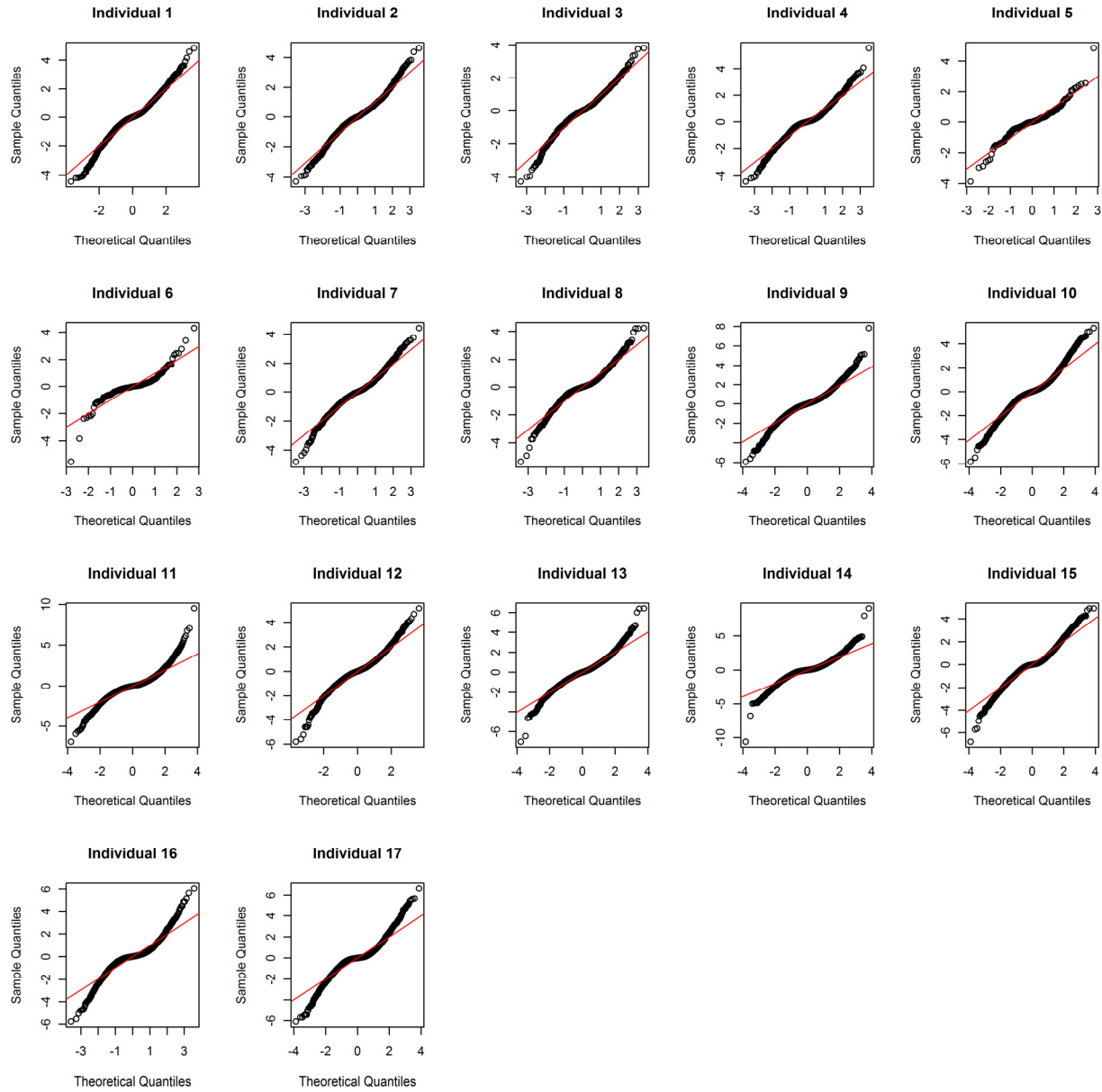


FIG. A5. Normal QQ plots for the posterior means of the observation error terms,  $\varepsilon_{x_{n,j,i}}$  (a) and  $\varepsilon_{y_{n,i}}$  (b) for individuals  $n = 1, \dots, 17$ .

(a)





(b)

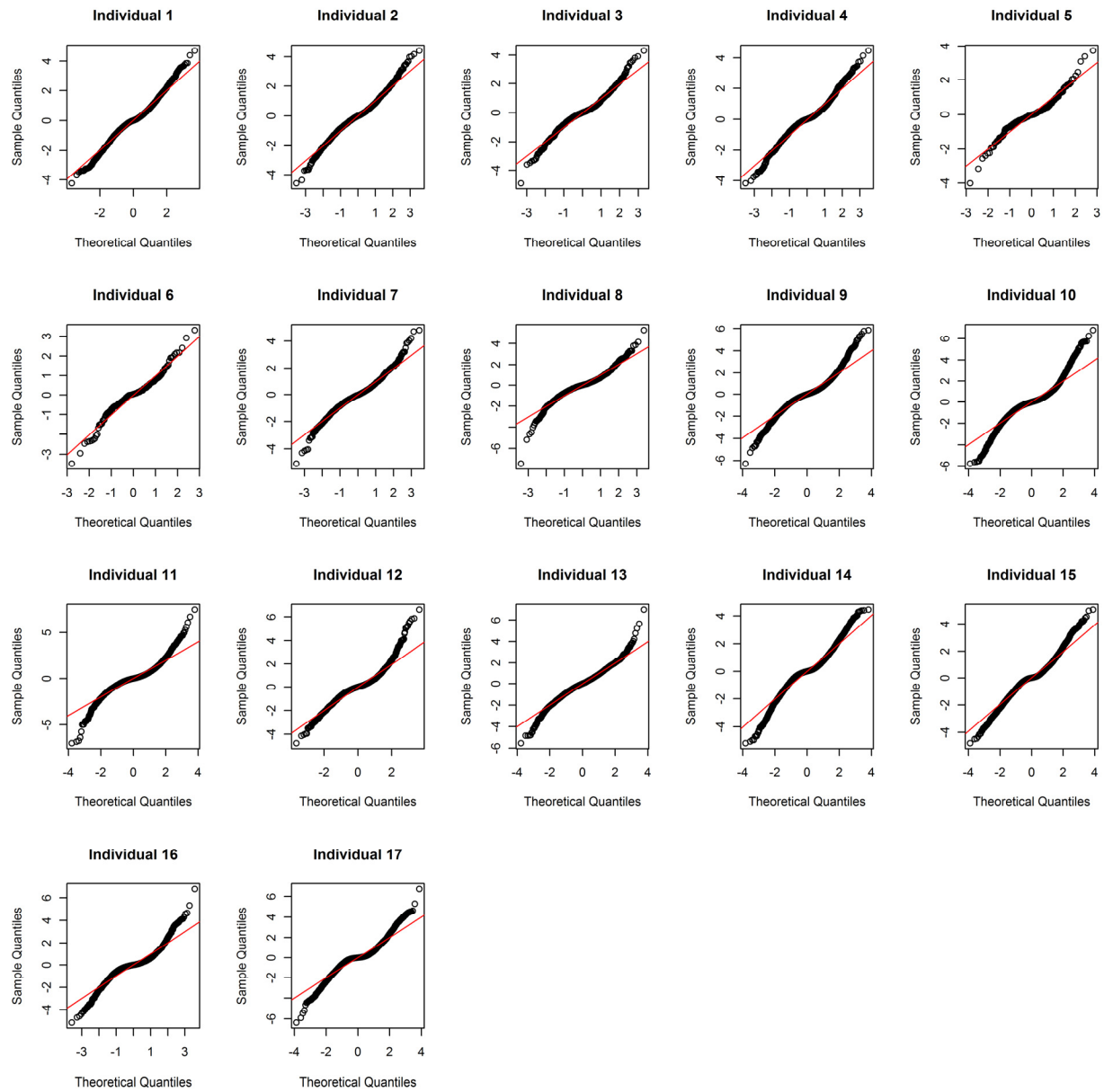
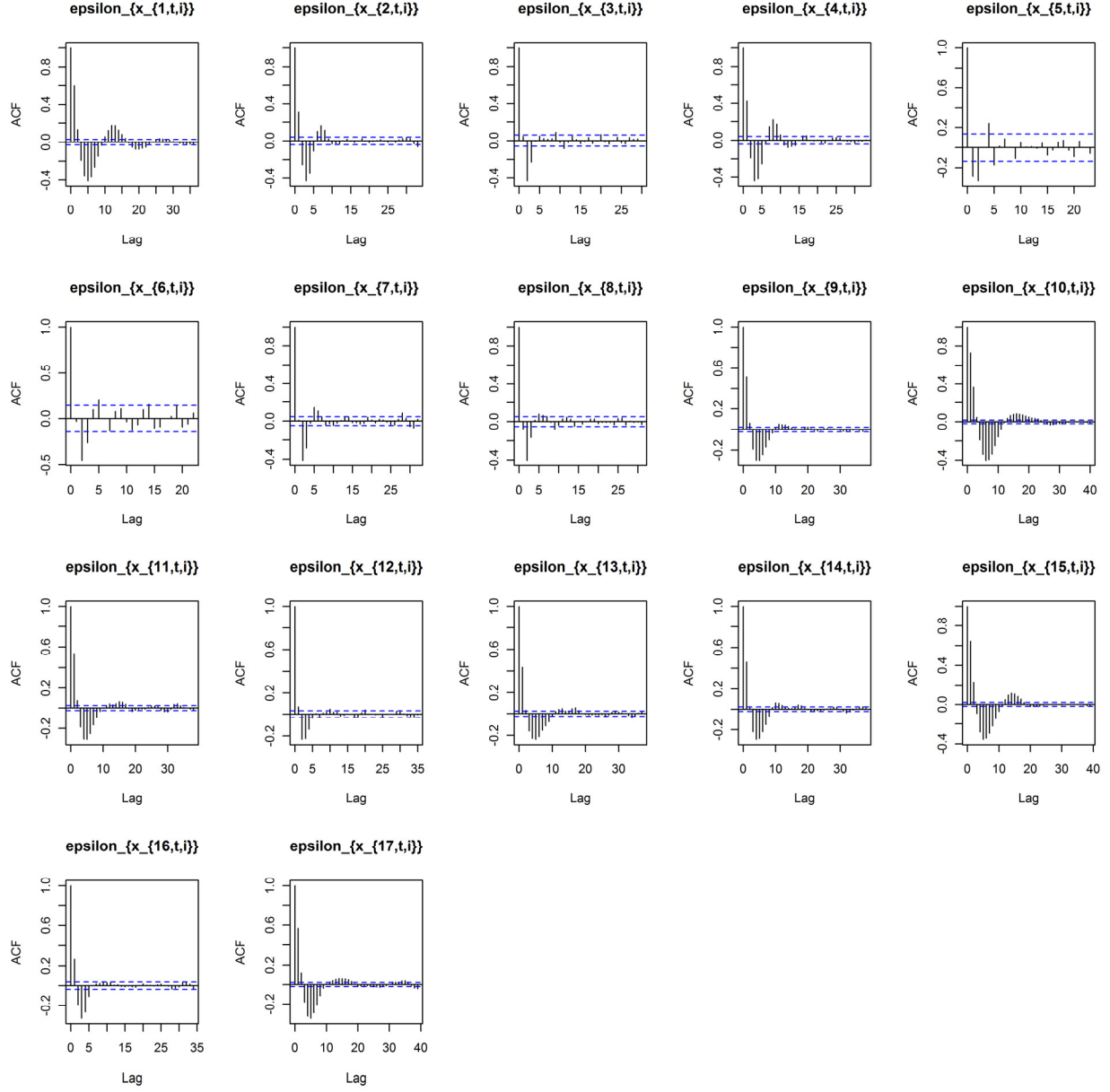


FIG. A6. Auto-correlation plots for the posterior means of the observation error terms,  $\varepsilon_{x_{n,j,i}}$  (a) and  $\varepsilon_{y_{n,j,i}}$  (b) for individuals  $n = 1, \dots, 17$ .

(a)



(b)

