


A tutorial to perform fourth-corner and RLQ analyses in

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Abstract

This tutorial shows how the fourth-corner and RLQ analyses can be performed using the `ade4` package (Dray and Dufour, 2007) for . The dataset presented in the paper is used to illustrate the methods. Commands are written in red and outputs are written in blue.

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1 Loading the package and the data

The different methods presented in the paper are available in the `ade4` package. The first step is to load `ade4` and the datasets:

```
library(ade4)
data(aravo)
```

The `aravo` dataset contains the abundances of 82 species in 75 sites. Species are described by 8 traits and 6 environmental variables have been measured in the sites.

```
dim(aravo$spe)
```

```
[1] 75 82
```

```
dim(aravo$traits)
```

```
[1] 82 8
```

```
dim(aravo$env)
```

```
[1] 75 6
```

2 RLQ

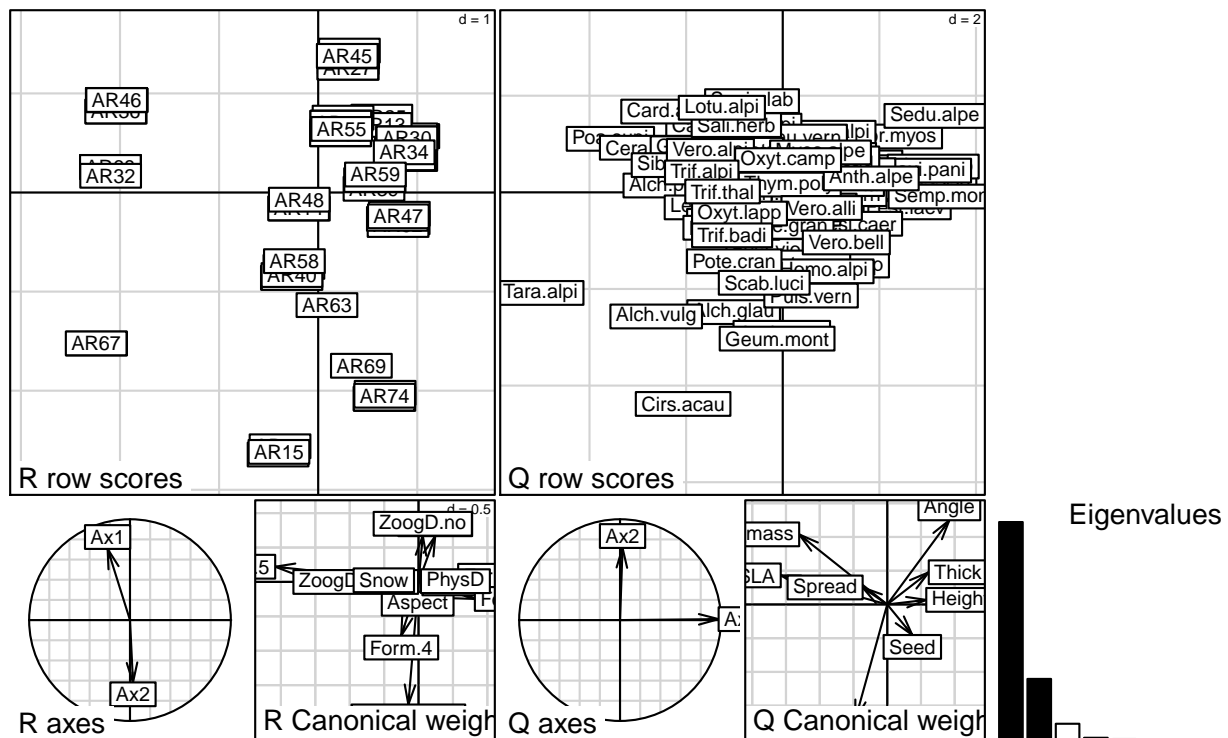
A preliminary step of RLQ analysis is to perform the separate analyses of each table. Correspondence analysis (`dudi.coa`) is applied to the species table. For traits data, all variables are quantitative and thus we applied a principal component analyses (`dudi.pca`). The environmental table contains both quantitative and categorical variables. In this case, we used the `dudi.hillsmith` function that allows to consider mix of different types of variables. Note that to proceed RLQ analysis, separate analyses of traits and environmental variables should be weighted by the sites and species weights derived from the previous correspondence analysis.

```
afcL.aravo <- dudi.coa(aravo$spe, scannf = FALSE)
acpR.aravo <- dudi.hillsmith(aravo$env, row.w = afcL.aravo$lw,
  scannf = FALSE)
acpQ.aravo <- dudi.pca(aravo$traits, row.w = afcL.aravo$cw,
  scannf = FALSE)
rlq.aravo <- rlq(acpR.aravo, afcL.aravo, acpQ.aravo,
  scannf = FALSE)
```

RLQ analysis finds coefficients (in `$c1`) to obtain a linear combination of traits (species scores in `$lQ`) and coefficients (in `$l1`) to obtain a linear combination of environmental variables (site scores in `$lR`). The covariance between these two sets scores is maximized and equal to the square root of the corresponding eigenvalue.

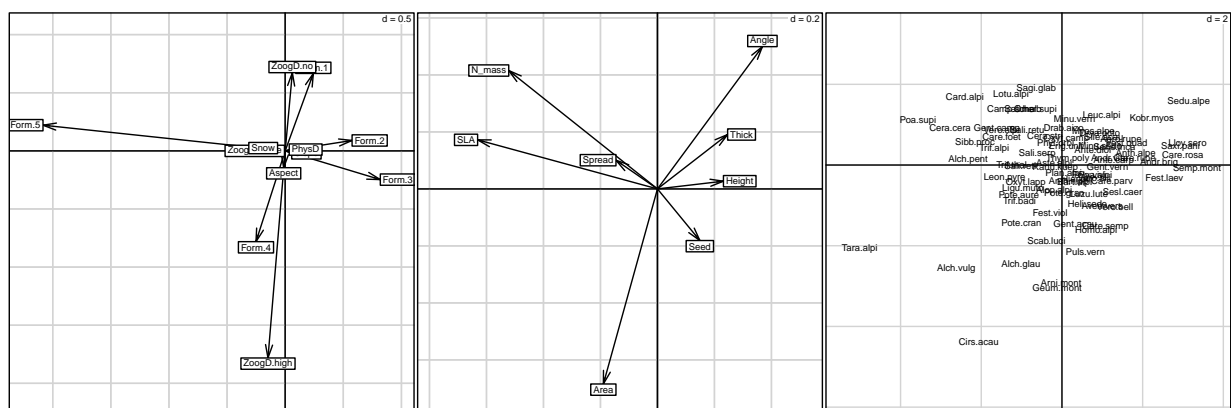
The different outputs of the analysis are obtained by the `plot` function:

```
plot(rlq.aravo)
```



The different figures can be obtained separately by plotting the different elements contained in the `rlq.aravo` object:

```
par(mfrow = c(1, 3))
s.arrow(rlq.aravo$11)
s.arrow(rlq.aravo$c1)
s.label(rlq.aravo$lQ, boxes = FALSE)
```



As RLQ analysis maximizes the covariance between the traits and the environmental variables mediated by the species abundances, it is important to see how the individual parts (i.e. $cov(traits, env)^2 = var(traits) \times var(env) \times cor(traits, env)^2$) of the compromise are considered. Hence, one can compare the RLQ analysis to the separate analyses which maximize independently the structure of the trait (principal component analysis of the traits), the structure of the environment (Hill-Smith analysis of the environmental variables) and the correlation (correspondence analysis of the sites-species table). These comparisons are provided by the `summary` function.

```
summary(rlq.aravo)

Eigenvalues decomposition:
  eig covar sdR sdQ corr
1 0.4280 0.6542 0.9924 1.527 0.4318
2 0.1198 0.3462 1.0991 1.145 0.2751

Inertia & coinertia R:
  inertia max ratio
1 0.985 1.321 0.7457
12 2.193 2.537 0.8645

Inertia & coinertia Q:
  inertia max ratio
1 2.331 2.409 0.9675
12 3.641 3.907 0.9319

Correlation L:
  corr max ratio
1 0.4318 0.8128 0.5312
2 0.2751 0.6484 0.4243
```

The correlation is quite low for the second axis (0.2753). The variance of the environmental scores is well preserved on the first two axes (89.53 %). For the traits, the amount of variance preserved (2.341 and 3.667) is nearly equal to the amount obtained in simple principal component analysis (2.409 and 3.907).

3 Fourth-corner

Fourth-corner analysis can be used to test the associations between individual traits and environmental variables. To obtain a test with a correct type I error, results of model 2 (permutation of sites, i.e. rows) and 4 (permutation of species, i.e. columns) should be combined. While the former version of `ade4` required three steps (tests with the two models and combination of the results), the combined approach can now be run by setting the `modeltype` argument to 6. Note that we used a very high number of repetitions (`nrepet <- 49999`) to have enough power in corrected tests. This is time-consuming and could be modified to speed up the different analyses (e.g., `nrepet <- 999`). Note that by default, the `nrepet` argument of the `fourthcorner` function is set to 999.

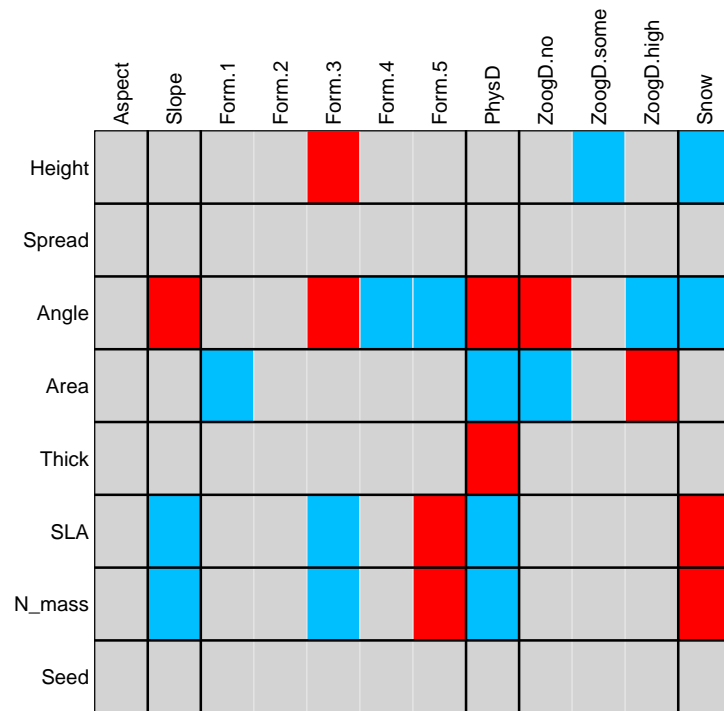
```
nrepet <- 49999
four.comb.aravo <- fourthcorner(aravo$env, aravo$spe,
  aravo$traits, modeltype = 6, p.adjust.method.G = "none",
  p.adjust.method.D = "none", nrepet = nrepet)
```

Then, results can be plotted. Blue cells correspond to negative significant relationships while red cells correspond to positive significant relationships (this can be modified using the argument `col`). In this example, there are some associations between categorical traits and quantitative environmental variable which can be measured in three different ways (Legendre et al., 1997). These three methods correspond to three possible values of the `stat` argument in the `plot` and `print` functions:

- `stat="D2"`: the association is measured between the quantitative variable and each category separately. A correlation coefficient is used to indicate the strength of the association between the given category and the small or large values of the quantitative variable.
- `stat="G"`: the association between the quantitative variable and the whole categorical variable is measured by a global statistic (F).
- `stat="D"`: the association is estimated between the quantitative variable and each category separately by a measure of the within-group homogeneity. The strength of the association is indicated by the dispersion of the values of the quantitative variable for a given category.

In the rest of the tutorial, we focus on the D2 statistic. The correction of p-values by a sequential procedure leads to significant associations if the maximal pvalue is lower than $\alpha = 0.05$. When $\alpha = 0.05$, there are only 26 significant associations (while there are 51 when $\alpha = \sqrt{0.05}$, i.e. the biased version proposed by Dray and Legendre (2008)).

```
plot(four.comb.aravo, alpha = 0.05, stat = "D2")
```



Now, we adjusted p-values for multiple comparisons (here we used the `fdr` method using the `p.adjust.4thcorner` function).

```
four.comb.aravo.adj <- p.adjust.4thcorner(four.comb.aravo,
  p.adjust.method.G = "fdr", p.adjust.method.D = "fdr")
```

Note that, adjusted p-values can be obtained directly using the `fourthcorner` function:

```
fourthcorner(aravo$env, aravo$spe, aravo$traits, modeltype = 6,
  p.adjust.method.G = "fdr", p.adjust.method.D = "fdr",
  nrepet = nrepet)
```

When adjusted p-values are used, there are 18 significant associations:

```
plot(four.comb.aravo.adj, alpha = 0.05, stat = "D2")
```

	Aspect	Slope	Form.1	Form.2	Form.3	Form.4	Form.5	PhysD	ZoogD.no	ZoogD.some	ZoogD.high	Show
Height												
Spread												
Angle												
Area												
Thick												
SLA												
N_mass												
Seed												

4 Combining both approaches

First, a multivariate test can be applied to evaluate the global significance of the traits-environment relationships. This test is based on the total inertia of the RLQ analysis :

```
testrlq.aravo <- randtest(rlq.aravo, modeltype = 6, nrepet = nrepet)
```

The results are highly significant:

```
testrlq.aravo
```

```
class: krandtest
Monte-Carlo tests
Call: randtest.rlq(xtest = rlq.aravo, nrepet = nrepet, modeltype = 6)
```

```
Number of tests: 2
```

```
Adjustment method for multiple comparisons: none
```

```
Permutation number: 49999
```

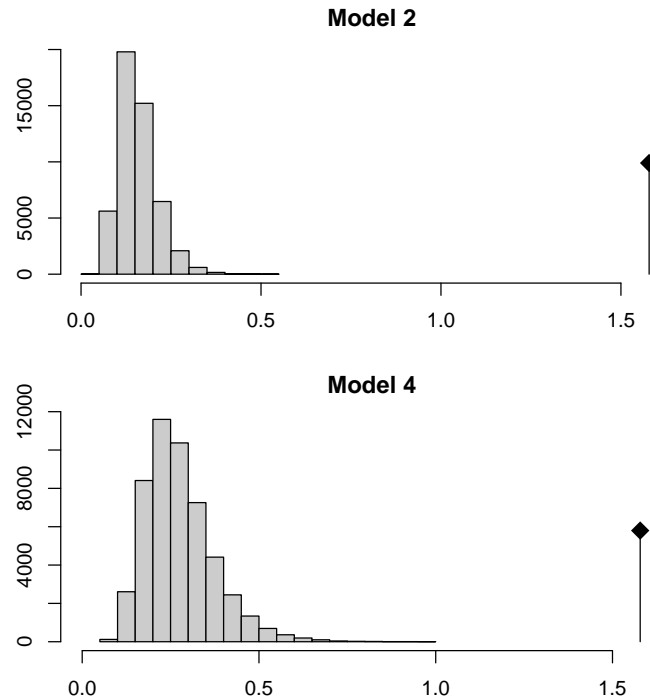
```
Test Obs Std.Obs Alter Pvalue
```

```
1 Model 2 1.578 26.79 greater 2e-05
```

```
2 Model 4 1.578 13.36 greater 2e-05
```

```
other elements: adj.method call comb.pvalue
```

```
plot(testrlq.aravo)
```



The total inertia of RLQ analysis is equal to the S_{RLQ} multivariate statistic defined in Dray and Legendre (2008). This statistic is returned by the `fourthcorner2` function:

```
Srlq <- fourthcorner2(aravo$env, aravo$spe, aravo$traits,
  modeltype = 6, p.adjust.method.G = "fdr", nrepet = nrepet)
```

```
Srlq$trRLQ
```

```
Monte-Carlo test
Call: fourthcorner2(tabR = aravo$env, tabL = aravo$spe, tabQ = aravo$traits,
  modeltype = 6, nrepet = nrepet, p.adjust.method.G = "fdr")
```

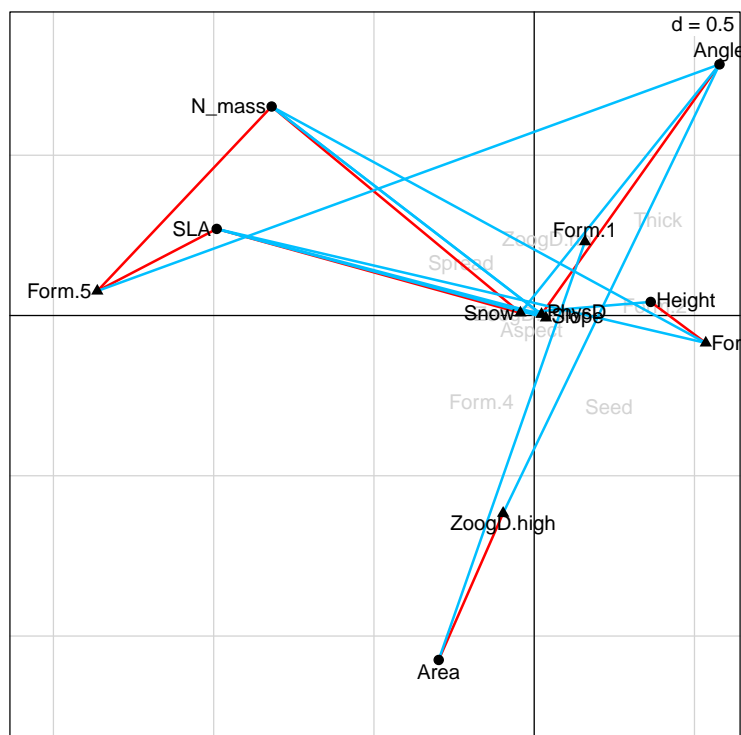
```
Observation: 1.578
```

```
Based on 49999 replicates
Simulated p-value: 2e-05
Alternative hypothesis: greater
```

Std.Obs	Expectation	Variance
26.739535	0.157577	0.002823

Both approaches can be combined if RLQ scores are used to represent traits and environmental variables on a biplot. Then, significant associations revealed by the `fourthcorner` approach can be represented using segments (blue lines for negative associations, red lines for positive associations, see the argument `col`). Only traits and environmental variables that have at least one significant association are represented. Here, we apply this method using adjusted pvalues for multiple comparisons and a significant level $\alpha = 0.05$.

```
plot(four.comb.aravo.adj, x.rlq = rlq.aravo, alpha = 0.05,
  stat = "D2", type = "biplot")
```



Another approach is provided by the `fourthcorner.rlq` function and consists in testing directly the links between RLQ axes and traits (`typetest="Q.axes"`) or environmental variables (`typetest="R.axes"`).

```
testQaxes.comb.aravo <- fourthcorner.rlq(rlq.aravo, modeltype = 6,
  typetest = "Q.axes", nrepet = nrepet, p.adjust.method.G = "fdr",
  p.adjust.method.D = "fdr")
testRaxes.comb.aravo <- fourthcorner.rlq(rlq.aravo, modeltype = 6,
  typetest = "R.axes", nrepet = nrepet, p.adjust.method.G = "fdr",
  p.adjust.method.D = "fdr")
```

```
print(testQaxes.comb.aravo, stat = "D")
```

Fourth-corner Statistics

Permutation method Comb. 2 and 4 (49999 permutations)

Adjustment method for multiple comparisons: fdr

call: `fourthcorner.rlq(xtest = rlq.aravo, nrepet = nrepet, modeltype = 6, typetest = "Q.axes", p.adjust.method.G = "fdr")`

	Test Stat	Obs	Std.Obs	Alter	Pvalue
1	AxcR1 / Height	r	0.166172	1.6580	two-sided 0.0969
2	AxcR2 / Height	r	0.005482	0.0974	two-sided 0.9247
3	AxcR1 / Spread	r	-0.095280	-0.9973	two-sided 0.32498
4	AxcR2 / Spread	r	0.034635	0.5099	two-sided 0.61842
5	AxcR1 / Angle	r	0.253857	2.5567	two-sided 0.00826
6	AxcR2 / Angle	r	0.152269	2.1152	two-sided 0.0327
7	AxcR1 / Area	r	-0.119618	-1.2661	two-sided 0.20862
8	AxcR2 / Area	r	-0.210731	-2.9267	two-sided 0.00278
9	AxcR1 / Thick	r	0.168446	1.6856	two-sided 0.092
10	AxcR2 / Thick	r	0.058806	0.8700	two-sided 0.39106
11	AxcR1 / SLA	r	-0.434786	-4.4161	two-sided 2e-05
12	AxcR2 / SLA	r	0.063480	0.8974	two-sided 0.37562
13	AxcR1 / N_mass	r	-0.360835	-3.6381	two-sided 6e-05
14	AxcR2 / N_mass	r	0.139740	1.9484	two-sided 0.04946
15	AxcR1 / Seed	r	0.107564	1.0502	two-sided 0.30206
16	AxcR2 / Seed	r	-0.059631	-0.8104	two-sided 0.42866
Pvalue.adj					
1					0.1938
2					0.9247
3	0.472698181818182				0.659648
4					0.03304 *
5					0.10464
6					0.37088
7					0.014826666666667 *
8					0.1938
9	0.481304615384615				0.00032 ***
10					0.481304615384615
11					0.00048 ***
12					
13					

```

14 0.131893333333333
15 0.472698181818182
16 0.489897142857143

```

```

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

```

print(testRaxes.comb.aravo, stat = "D")

```

Fourth-corner Statistics

```

-----
Permutation method Comb. 2 and 4 ( 49999 permutations)

```

```

Adjustment method for multiple comparisons:  fdr

```

```

call:  fourthcorner.rlq(xtest = rlq.aravo, nrepet = nrepet, modeltype = 6,      typetest = "R.axes", p.adjust.method.G = "fdr")

```

```

---

```

	Test	Stat	Obs	Std.Obs	Alter	Pvalue
1	Aspect / AxcQ1	r	-0.01259	-0.2301	two-sided	0.8181
2	Slope / AxcQ1	r	0.22652	4.1595	two-sided	2e-05
3	Form.1 / AxcQ1	Homog.	0.23597	-0.1475	less	0.4639
4	Form.2 / AxcQ1	Homog.	0.08358	-0.3476	less	0.42034
5	Form.3 / AxcQ1	Homog.	0.26129	-0.9018	less	0.19488
6	Form.4 / AxcQ1	Homog.	0.10294	-1.6170	less	0.01574
7	Form.5 / AxcQ1	Homog.	0.13521	-0.5296	less	0.32994
8	PhysD / AxcQ1	r	0.30944	5.6603	two-sided	2e-05
9	ZoogD.no / AxcQ1	Homog.	0.56986	2.8684	less	0.99978
10	ZoogD.some / AxcQ1	Homog.	0.30680	-1.5000	less	0.05672
11	ZoogD.high / AxcQ1	Homog.	0.12234	-1.3253	less	0.05572
12	Snow / AxcQ1	r	-0.49740	-9.1299	two-sided	2e-05
13	Aspect / AxcQ2	r	-0.09833	-2.2865	two-sided	0.02102
14	Slope / AxcQ2	r	-0.06159	-1.2263	two-sided	0.22278
15	Form.1 / AxcQ2	Homog.	0.17597	-1.5871	less	0.0399
16	Form.2 / AxcQ2	Homog.	0.09112	-0.0249	less	0.55324
17	Form.3 / AxcQ2	Homog.	0.28641	-0.1094	less	0.48694
18	Form.4 / AxcQ2	Homog.	0.18260	0.8655	less	0.80482
19	Form.5 / AxcQ2	Homog.	0.24224	1.9512	less	0.9591
20	PhysD / AxcQ2	r	0.06799	1.1106	two-sided	0.27244
21	ZoogD.no / AxcQ2	Homog.	0.31611	-2.6342	less	0.0018
22	ZoogD.some / AxcQ2	Homog.	0.37582	0.1249	less	0.56468
23	ZoogD.high / AxcQ2	Homog.	0.24031	2.0419	less	0.9611
24	Snow / AxcQ2	r	0.12693	1.2298	two-sided	0.2239
Pvalue.adj						
1	0.908356363636364					
2	0.00016	***				
3	0.618533333333333					
4	0.63051					
5	0.33408					
6	0.0539657142857143	.				
7	0.465797647058824					
8	0.00016	***				
9	0.99978					
10	0.123752727272727					
11	0.121570909090909					
12	0.00016	***				
13	0.056053333333333	.				
14	0.35824					
15	0.09576	.				
16	0.69882947368421					
17	0.687444705882353					
18	0.919794285714286					
19	0.99378					
20	0.40866					
21	0.00864	**				
22	0.71328					
23	0.99978					
24	0.35824					

```

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

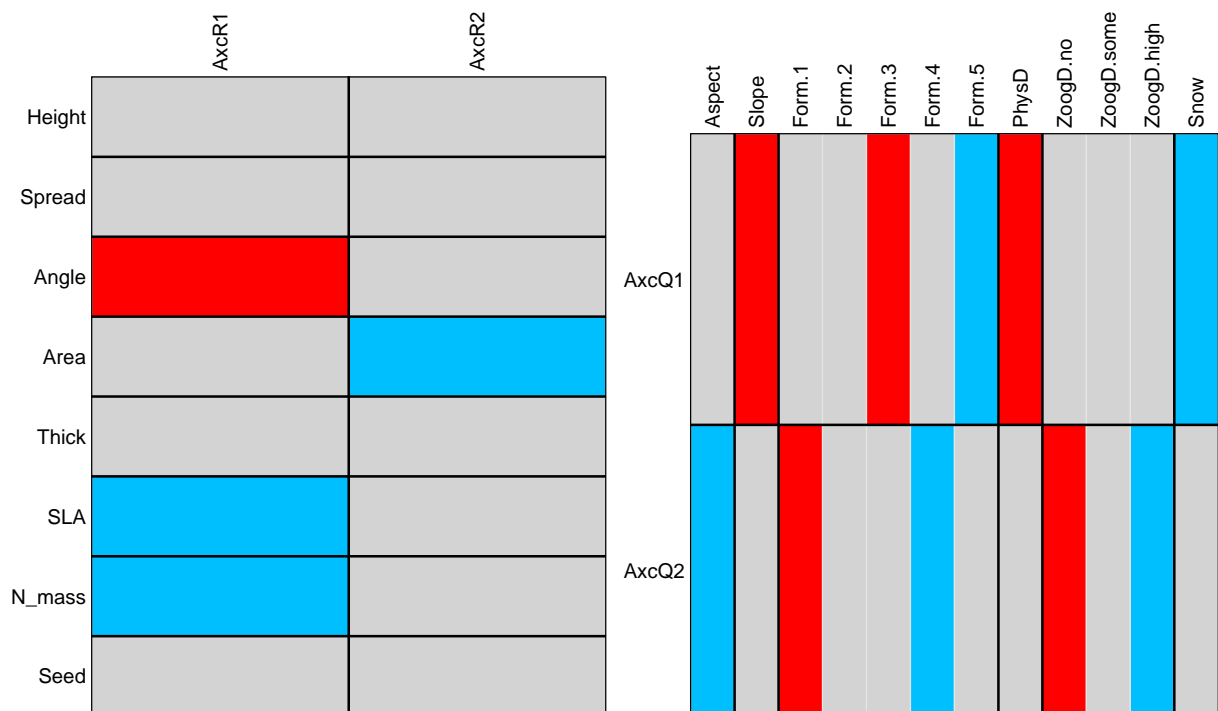
```

Results can be represented using a table with colors indicating significance :

```

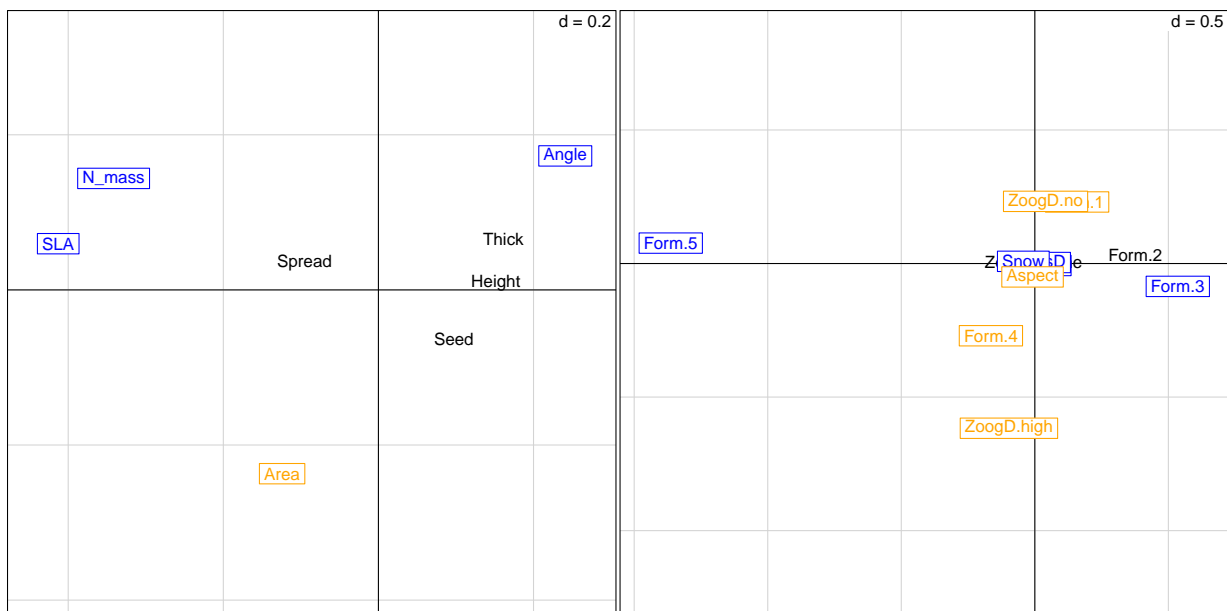
par(mfrow = c(1, 2))
plot(testQaxes.comb.aravo, alpha = 0.05, type = "table",
     stat = "D2")
plot(testRaxes.comb.aravo, alpha = 0.05, type = "table",
     stat = "D2")

```



Significance with axes can also be reported on the factorial map of RLQ analysis. Here, significant associations with the first axis are represented in blue, with the second axis in orange, with both axes in green (variables with no significant association are in black):

```
par(mfrow = c(1, 2))
plot(testQaxes.comb.aravo, alpha = 0.05, type = "biplot",
     stat = "D2", col = c("black", "blue", "orange", "green"))
plot(testRaxes.comb.aravo, alpha = 0.05, type = "biplot",
     stat = "D2", col = c("black", "blue", "orange", "green"))
```



References

S. Dray and A. B. Dufour. The ade4 package: implementing the duality diagram for ecologists. *Journal of Statistical Software*, 22(4):1–20, 2007.

- S. Dray and P. Legendre. Testing the species traits-environment relationships: the fourth-corner problem revisited. *Ecology*, 89:3400–3412, 2008.
- P. Legendre, R. Galzin, and M. L. Harmelin-Vivien. Relating behavior to habitat: solutions to the fourth-corner problem. *Ecology*, 78(2):547–562, 1997.