APPENDIX B: ADDITIONAL DETAILS ON ENERGETICS AND MODELING

Optimal lactation

Several of the choices outlined in Appendix A required that the optimal amount of milk energy transferred from mother to pup was also assessed. The resolution with which optimal energy transfers were assessed had the greatest impact on the running time of computations. We used a two-stage search process with a resolution of five at each stage. For example, consider the calculations in Eq. A.13. If the female has a maximum energy storage capacity of 4,000MJ and a current store of $x = 0.3$, then 1,200MJ are available to transfer to the pup. Given a daily maximum energy transfer of $L_{max} = 135$MJ, then the amount transferred is obviously limited by this latter constraint. In this case, we compute Eq. A.13 using $L = 0, 0.2L_{max}, 0.4L_{max} \ldots L_{max}$. The value of L at which the payoff is maximized is recorded as zL, and the process is repeated for $L = (z - 0.2)L_{max} \ldots (z + 0.2)L_{max}$. Note, however, that L is still bounded by zero and L_{max}. This suggests that the best amount of energy to be transferred to the pup can be determined to the nearest 10.8MJ, which represents the nearest 0.27% of potential maternal stores.

Grid interpolation

As we noted above, computations are based on a discrete grid of values for the x and y state variables. Behaviour results in a transition from (x,a,y,f,t) to (x',a',y',f',t'). For the x and y state variables this requires interpolation. Specifically, if g is the size of intervals on the discrete grid, we define the function trunc(z), to return the closest multiple of g for which trunc(z) $\leq z$. We then define

$$x_1 = \text{trunc}(x') + g$$
$$x_2 = \text{trunc}(x')$$
$$r = x' - x_2.$$

and

$$y_1 = \text{trunc}(y') + g$$
$$y_2 = \text{trunc}(y')$$
$$s = y' - y_2.$$

The value of a female in condition (x',a',y',f',t') is then estimated as:

$$V(x',a',y',f',t') = rs \cdot V(x_1,a',y_1,f',t') + r(1\text{-}s) \cdot V(x_1,a',y_2,f',t') + s(1\text{-}r) \cdot V(x_2,a',y_1,f',t') + (1\text{-}r)(1\text{-}s) \cdot V(x_2,a',y_2,f',t') \qquad \text{Eq. B.1}$$

Determining the value of independent young

The value of young released at time t and with reserves y is obviously an infinitely recursive concept, because the fitness of these animals depends on the value of the young they will have in the future. Thus, it is necessary to use a proxy for the value of independent young, $R(y,t)$. For this, we use the probability that the pup will survive to reach lean adult mass, a value determined using a separate submodel, as follows.

Independent pups are characterised by four state variables:

h lean mass, as a proportion of adult lean mass;
j fat stores as a proportion of adult maximum stores;
d days since independence;
t day of the year.

At independence, y dependent pup reserves are allocated to the h and j variables of the independent pup. Based on data in Muelbert et al. (2003), we assume that pup mass at weaning (in kg) is approximately 1/20th of total body energy (in MJ), and that fat stores at weaning represent approximately 80% of total body energy. This allows us to relate stored reserves of a pup receiving maternal care to both lean mass and fat reserves at weaning. The value of pups for which y falls between grid values of h and j is calculated by interpolation among the values of pups with neighbouring h and j.

To determine the value of independent pups at time t with lean mass h and fat reserves j, the same backwards iteration approach described for the main model is used. On each day, independent pups choose to forage or rest. If they rest, their metabolism is 2.2BMR estimated for an animal of their lean mass (calculated in the same way as for adults but for different lean body sizes). If they forage, their metabolism is BMR($b + jc$) where b and c are the same constants used to estimate mass dependent costs for adults (see Table 3 in the main text), and BMR is again based on expected metabolism given the pup's size.

Foraging independent pups gain food given the same environmental parameters faced by adults, as described in the main text. Metabolised food energy gained [$E = q_{rel}.e(t)/p$ with probability p, or zero with probability $1 - p$] is allocated to lean growth and fat reserves in proportions z and $1 - z$, respectively [z is varied over a coarse grid, where $z \in (0.0, 0.33, 0.67, 1.0)$]. Throughout the independent pup period, daily survival is interpolated between dependent pup survival and adult survival, according to the proportion of adult size reached. If reserves drop to $j = 0$ the pup dies. An absorbing boundary [with $V(h,j,d,t) = 1, j \neq 0$] for $h = 1$, ensures that full adult lean size ($h = 1$) must be reached for the pup to attain value.

LITERATURE CITED

Muelbert, M. M. C., W. D. Bowen, and S. J. Iverson. 2003. Weaning mass affects changes in body composition and food intake in harbour seal pups during the first month of independence. Physiological and Biochemical Zoology 76**:**418–427.