Ecological Archives ###

Michael Kalyuzhny, Yishai Schreiber, Rachel Chocron, Curtis H. Flather, Ronen Kadmon, David A. Kessler, and Nadav M. Shnerb. 2014. Temporal fluctuation scaling in populations and communities. *Ecology* ####.

Appendix C. The alpha-beta-gamma model (see Fig. 2 in main text).

For a population starting with n_0 individuals, we assume that the chance of an individual remaining "inactive" (not involved in a birth or death event) until the next census is α. Accordingly, the number of inactive individuals k is taken from a binomial distribution,

$$P(k) = \binom{n_0}{k} \alpha^k (1-\alpha)^{n_0-k}.$$

An active individual dies with probability 1- β and give birth with probability β. To keep the population fixed (on average), every reproducing individual must produce on average 1/ β offspring. If, due to environmental noise, the population is growing or shrinking, the average number of offspring is (1+ γ)/ β. We assume that γ is taken from a uniform distribution with zero mean and variance Δ.

The actual number of offspring are taken to be integers drown from a Poisson distribution with mean (1+ γ)/ β. Therefore, the chance to census n_1 individuals in the next census, given n_0 in the current census, is

$$P(n_1|n_0) = \sum_{k=0}^{n_0} \binom{n_0}{k} \alpha^k (1-\alpha)^{n_0-k} \sum_{s=0}^{n_0-k} \binom{n_0-k}{s} \beta^s (1-\beta)^{n_0-k-s} \, \frac{e^{-s\frac{(1+\gamma)}{\beta}} \left[s\frac{(1+\gamma)}{\beta}\right]^{n_1-k}}{(n_1-k)!}$$

The average number in the next census, $\overline{n_1} = \sum_{n_1=0}^{\infty} n_1 P(n_1|n_0)$, may be calculated using

$$\sum_{n_1=k}^{\infty} n_1 \frac{\rho^{n_1-k}}{(n_1-k)!} = \rho \frac{\partial}{\partial \rho} \sum_{n_1-k=0}^{\infty} \frac{\rho^{n_1-k}}{(n_1-k)!} + k \sum_{n_1-k=0}^{\infty} \frac{\rho^{n_1-k}}{(n_1-k)!} = \rho \frac{\partial}{\partial \rho} e^{\rho} + k e^{\rho}$$

Where $s\frac{(1+\gamma)}{\beta} = \rho$. The result

$$\overline{n_1} = n_0\alpha + n_0(1-\alpha)(1+\gamma),$$

is as expected. Similarly

$$var\ (n_1) = n_0(1-\alpha)\left[\frac{(1+\gamma)^2}{\beta} - \gamma(1+\gamma) + \gamma^2\alpha\right]$$

Since $\overline{\gamma} = 0$,

$$var\ (n_1) = \frac{n_0(1-\alpha)}{\beta} + \Delta n_0\left[\frac{(1-\alpha)}{\beta} - (1-\alpha)^2\right] + \Delta n_0^2(1-\alpha)^2.$$

Defining the rescaled amplitude of fluctuation $Y = \frac{n_1 - n_0}{\sqrt{n_0}}$, its variance $Var\ (Y) = \frac{var\ (n_1)}{n_0}$ is given by Eq. (3) of the main text. **Thus, for any process without environmental noise (no matter what the strength of demographic stochasticity β) Var(*Y*) will be independent of the initial population size.**