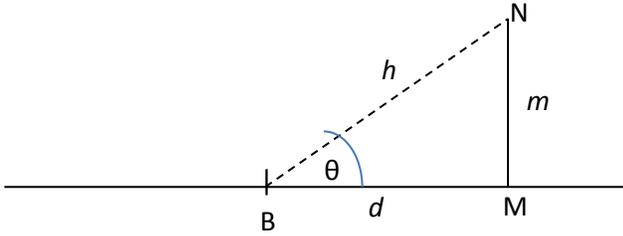


**Appendix A.** Mathematical rationale of the tradeoff function of branching angles that yields the maximum payoff.



Consider a new food source located at a site N near a main foraging trail (horizontal line). A straight perpendicular line (at 90°) would be the minimum distance  $m$  that connects N with the main trail. Let  $d$  be the distance between a bifurcation point B and the intersection point M of the minimum distance  $m$ . The additional distance that the ants build and maintain when using angles lower than 90° is  $\sqrt{m^2 + d^2}$ , or  $h$  the hypotenuse of the triangle. Then,  $b(d) = m + d - h$  is an efficiency function (Equation 1) that measures the net benefit (in distance units) of building at a certain angle compared to a perpendicular branch  $m$ . Similarly, we can calculate a function for the cost of maintaining a larger new trail section at a certain angle relative to the minimum length of the new trail (the perpendicular branch  $m$ ) as  $c(d) = h - m$  (Equation 2). This function measures the additional distance built (in distance units) compared to the minimum trail length they could have built at 90°.

A tradeoff function takes the traveling benefits (Equation 1) and subtracts the maintenance costs (Equation 2) by a factor  $\alpha$ , that weights how important are the costs relative to the benefits in determining the branching angle:  $f(d) = b(d) - \alpha * c(d) = m + d - h - \alpha * (h - m) = m + d - h - \alpha * h + \alpha * m$   
 $= m * (1 + \alpha) - h * (1 + \alpha) + d = (m - h) * (1 + \alpha) + d$

$$= (m - \sqrt{m^2 + d^2}) * (1 + \alpha) + d \text{ (Equation 3).}$$

The distance  $d$  and the corresponding angle where the function reaches its maximum would be the angle with the minimal maintenance costs and the largest traveling benefits. Now, it is simple to find the distance  $d$  that maximizes the net benefit:

$$f'(d) = 1 - \frac{(1 + \alpha) * d}{\sqrt{m^2 + d^2}} = 0$$

$$(1 + \alpha) * d = \sqrt{m^2 + d^2} \rightarrow (1 + \alpha)^2 * d^2 = m^2 + d^2 \rightarrow d^2 * [(1 + \alpha)^2 - 1] = m^2$$

$$d = \frac{m}{\sqrt{(1 + \alpha)^2 - 1}}$$

Therefore, if  $\alpha \rightarrow 0$ ,  $d \rightarrow \infty$  and if  $\alpha \rightarrow \infty$ ,  $d \rightarrow 0$ . To estimate the branching angles ( $\theta$ ) corresponding to intermediate values of  $\alpha$ ;

$$\theta = \text{atan}\left(\frac{m}{d}\right) = \text{atan}\left(\frac{m}{\frac{m}{\sqrt{(1 + \alpha)^2 - 1}}}\right) = \text{atan}\left(\sqrt{(1 + \alpha)^2 - 1}\right)$$

This tradeoff function predicts a branching angle of  $60^\circ$  when maintenance costs and shortening travel distances are of equal importance (i.e.,  $\alpha = 1$ ); branching angles  $< 60^\circ$  maximize the function when trail maintenance costs are relatively less important than shortening travel distances (i.e.,  $\alpha < 1$ ), and branching angles  $> 60^\circ$  yield the maximum payoff when trail maintenance costs are relatively more important than shortening travel distances (i.e.,  $\alpha > 1$ ; see Fig. 2).