

C Autocorrelation estimation

Autocorrelated kernel-density estimation is contingent upon being able to first estimate the autocorrelation function (ACF, Eq. B.18) or, equivalently, the covariance and semi-variance function (SVF, Eq. B.33). A spatially-constrained autocorrelation model must be fit to the data and the data must suggest evidence of range-resident or territorial behavior. The first step to determining whether or not home-range behavior has been observed is to visually inspect the asymptotic behavior of the semi-variance function (SVF), as in Fig. C.1. The SVF is defined as the square distance an animal travels, on average, over a specified period of time. For stationary processes, the SVF is equivalent to the more familiar net-squared or mean-squared displacement. If the animal has a finite covariance, which corresponds to spatially limited movement and thus home-range behavior, then the SVF will asymptote to this covariance over some timescale that characterizes the persistence of autocorrelation. For our gazelle example in Fig. 2, there was visible evidence of a home range noted in Fleming et al. (2014a) Fig. 3.

Although, the presence of a home range can often be visually noted in the SVF estimate and rough parameter estimates can be obtained via variogram regression, maximum-likelihood analysis is the more rigorous method for parameter estimation and model comparison (Fleming and Calabrese, 2013; Fleming et al., 2014b). A likelihood ratio test or AIC comparison can be used to determine if there is support in the data for spatial constrained movement. If the evidence for spatial constraint is weak, then the corresponding large uncertainty in the spatial constraining parameters can be propagated into the AKDE estimate (App. B.3). Importantly, AKDE estimation in cases where it is not warranted will either fail outright or will yield very wide confidence intervals that appropriately reflect the lack of information on home range or territory size in the data.

In the hypothetical case of a canid that patrols its territory once per day, the longer autocorrelation timescale would be roughly given by $\tau_A \sim 1$ day. If the data spans many days, then the semi-variance function would appear similar to Fig. C.1 B, and AKDE would be appropriate. However, if the period of the data were only a few hours, then the semi-variance function would appear similar to Fig. C.1 A. Therefore, at this stage the researcher should not proceed to an AKDE analysis, or any home-range analysis, as the data will be visually inappropriate. However, let us assume that the researcher proceeds to estimate the autocorrelation parameters anyhow. If they perform a model comparison analysis, as we advocate in Fleming et al. (2014b), then spatially-constrained models such as Ornstein-Uhlenbeck motion should not be supported by the data over spatially-unconstrained models such as Brownian motion. However, let us assume that the researcher fails to perform model selection. Then, as the data do not suggest any spatial constraint, the spatially-constraining autocorrelation parameters will be estimated to have extremely large confidence intervals. Propagating these uncertainties into the AKDE will produce an area estimate with confidence intervals that are larger than the point estimate.

C.1 The importance of positive-definite estimators

The optimal bandwidth relations are derived under the assumption of a positive-definite autocorrelation function, and so applying an unconstrained estimate of the ACF to the MISE

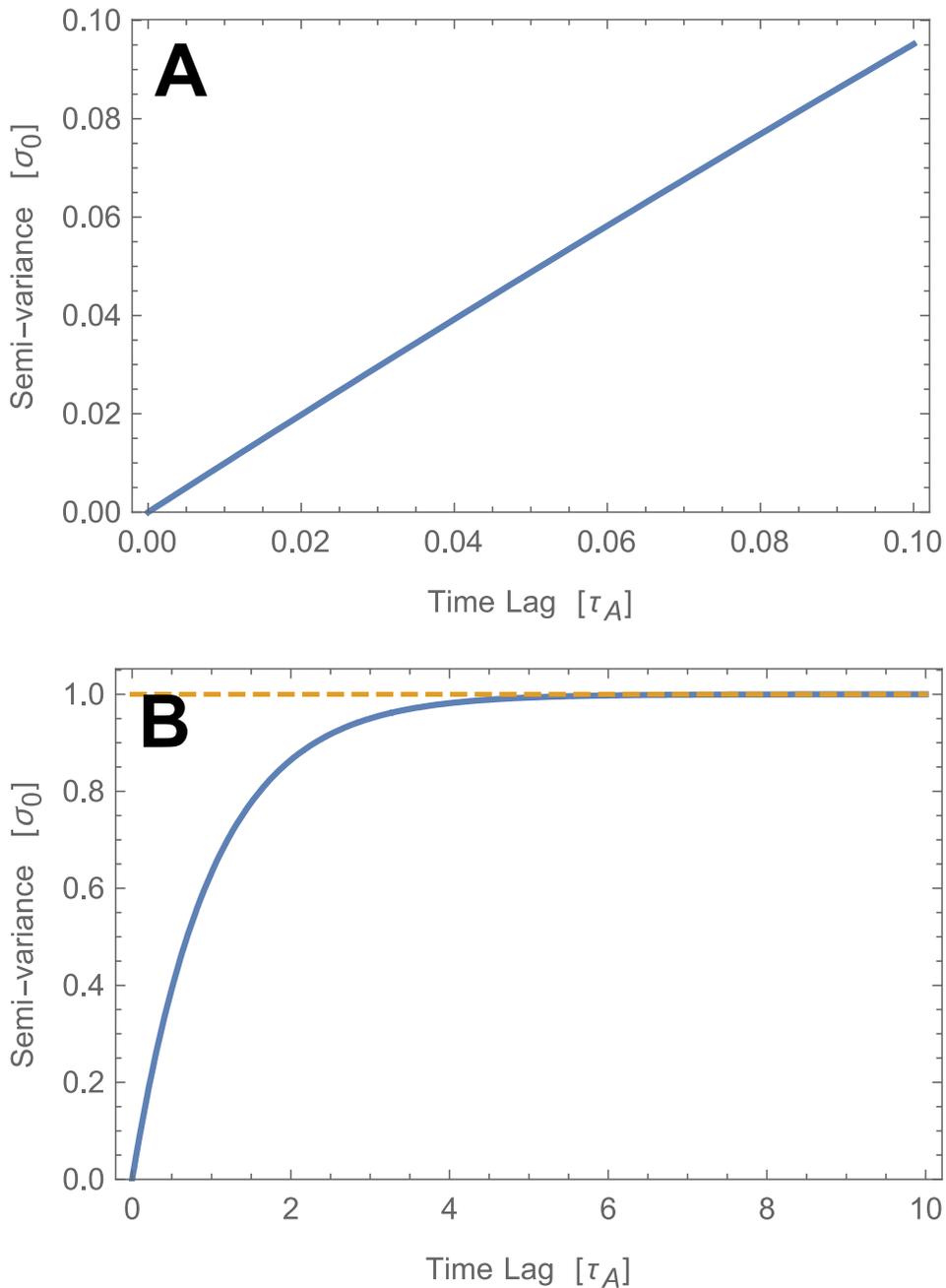


Figure C.1: **A:** A theoretical semi-variance function that is inappropriate for home-range estimation. In this example, it is impossible to see the SVF asymptote to a finite variance σ_0 nor over what timescale τ_A this would occur. **B:** The same semi-variance function (thick line) and asymptotic variance (dashed), but observed over a longer period of time so as to be appropriate for home-range estimation. Here it is clear that the variance and autocorrelation timescale can be estimated from the data.

can yield invalid results. Note that the stationary MISE (B.35) can be represented as

$$\varepsilon(\boldsymbol{\sigma}_B) = \frac{1}{(2\pi)^{\frac{q}{2}}} \left(\frac{1}{n^2} \sum_{\tau} \frac{n(\tau)}{\sqrt{\det(2\boldsymbol{\sigma}_0 - 2\boldsymbol{\sigma}(\tau) + 2\boldsymbol{\sigma}_B)}} - \frac{2}{\sqrt{\det(2\boldsymbol{\sigma}_0 + \boldsymbol{\sigma}_B)}} + \frac{1}{\sqrt{\det(2\boldsymbol{\sigma}_0)}} \right), \quad (\text{C.1})$$

If it is ever the case that $\boldsymbol{\sigma}(\tau) > \boldsymbol{\sigma}_0$, then the matrix sum in the first term can be indefinite, resulting in a negative or infinite MISE. For a positive-definite ACF, it is always the case that $-\boldsymbol{\sigma}_0 \leq \boldsymbol{\sigma}(\tau) \leq +\boldsymbol{\sigma}_0$ and so this will not happen. However, this bound alone is still insufficient. Consider an indefinite ACF estimate that takes the value $\boldsymbol{\sigma}(\tau) = -\boldsymbol{\sigma}_0$ for all $\tau \neq 0$. The MISE resolves to

$$\varepsilon(\boldsymbol{\sigma}_B) = \frac{1}{(2\pi)^{\frac{q}{2}}} \left(\frac{\frac{1}{n}}{\sqrt{\det(2\boldsymbol{\sigma}_B)}} + \frac{\frac{n-1}{n}}{\sqrt{\det(4\boldsymbol{\sigma}_0 + 2\boldsymbol{\sigma}_B)}} - \frac{2}{\sqrt{\det(2\boldsymbol{\sigma}_0 + \boldsymbol{\sigma}_B)}} + \frac{1}{\sqrt{\det(2\boldsymbol{\sigma}_0)}} \right). \quad (\text{C.2})$$

In the limit of large n and small bandwidth, the MISE is given by

$$\lim_{\boldsymbol{\sigma}_B \rightarrow \mathbf{0}} \lim_{n \rightarrow \infty} \varepsilon(\boldsymbol{\sigma}_B) = \frac{1}{(2\pi)^{\frac{q}{2}}} \frac{2^{-\frac{q}{2}} - 2 + 1}{\sqrt{\det(2\boldsymbol{\sigma}_0)}}, \quad (\text{C.3})$$

which is negative.

References

- Fleming, C. H., and J. M. Calabrese. 2013. On the estimators of autocorrelation model parameters ArXiv:1301.4968 [stat.ME].
- Fleming, C. H., J. M. Calabrese, T. Mueller, K. A. Olson, P. Leimgruber, and W. F. Fagan. 2014*a*. From fine-scale foraging to home ranges: A semi-variance approach to identifying movement modes across spatiotemporal scales. *The American Naturalist* 183:E154–E167.
- . 2014*b*. Non-Markovian maximum likelihood estimation of autocorrelated movement processes. *Methods in Ecology and Evolution* 5:462–472.