Appendix C – JAGS implementation of the spatial Gompertz model using the recursive parameterization

Predictive Gaussian modeling

For illustration purposes, we demonstrate how to implement the spatial Gompertz model using JAGS (Plummer 2003). For this purpose, we use the recursive parameterization of the model (Eq. 2a–2d). However, modeling abundance at each sampling station becomes computationally prohibitive as the number of sampling stations increases in number. We therefore adopt a predictive Gaussian approximation, wherein we model local densities at several (in this case 25) pre-defined locations (termed "knots"), and then predict local densities at each station given the current estimate of density at the knots whenever necessary.

Specifically, we define J knots, where knot j is at location $<x_j, y_j>$. $\mathbf{\Omega}$ is then approximated as a vector $\mathbf{\Omega}^{(j)} = <\omega_1, \omega_2, ..., \omega_J>$ representing productivity at each knot, while $\mathbf{E}_t$ is approximated as a vector $\mathbf{E}_t^{(j)} = <e_1, e_2, ..., e_J>$ representing unexplained variation in local density at each knots. These both follow a multivariate normal distribution:

$$\mathbf{\Omega}^{(j)} \sim MVN(0, \mathbf{\Sigma}_\Omega^{(jj)})$$
$$\mathbf{E}_t^{(j)} \sim MVN(0, \mathbf{\Sigma}_E^{(jj)})$$

(C.1)

where covariation among knots depends upon the distance $h(j,j')$ between knot j and j':

$$\mathbf{\Sigma}_\Omega^{(jj)} = \sigma_\Omega^2 \mathbf{R}_h(j, j')$$
$$\mathbf{\Sigma}_E^{(jj)} = \sigma_E^2 \mathbf{R}_h(j, j')$$

(C.2)

and parameters are as defined in the main text. Population dynamics is tracked at each knot, and is updated using Eq. 3a from the main text.

However, calculating the probability of data similarly requires calculating $\mathbf{E}_t$ and $\mathbf{\Omega}$ at each station:

$$\mathbf{\Omega}^{(i)} = \mathbf{\Sigma}_{\Omega}^{(ij)} \left(\mathbf{\Sigma}_{\Omega}^{(jj)} \right)^{-1} \mathbf{\Omega}^{(j)}$$

$$\mathbf{E}_{t}^{(i)} = \mathbf{\Sigma}_{E}^{(ij)} \left(\mathbf{\Sigma}_{E}^{(jj)-1} \right) \mathbf{E}_{t}^{(j)}$$

(C.3)

which in turn depends upon the covariation between knots 1 through J and stations 1 through I:

$$\mathbf{\Sigma}_{\Omega}^{(ij)} = \sigma_{\Omega}^{2} \mathbf{R}_{h}(i, j)$$

$$\mathbf{\Sigma}_{E}^{(ij)} = \sigma_{E}^{2} \mathbf{R}_{h}(i, j)$$

(C.4)

where $\mathbf{R}_h(i,j)$ is a matrix of $r_h(i,j)$, giving the correlation between station i and knot j given their

the distance $h(i,j)$. $\mathbf{\Psi}^{(i)}$ and $\mathbf{\Omega}^{(i)}$ are then used to calculate $\log(\mathbf{D}^{(i)}_{t})$ for all stations i and years t.

This predictive Gaussian formulation requires Markov chain Monte Carlo sampling for $\mathbf{\Omega}^{(j)}$ and

$\mathbf{E}_{t}^{(j)}$, in addition to parameters α, σ_{Ω}, σ_{Ψ}, ρ, and the parameter governing the range of correlation

κ.

LITERATURE CITED

Plummer, M. 2003. JAGS: A program for analysis of Bayesian graphical models using Gibbs

    sampling. Proceedings of the 3rd International Workshop on Distributed Statistical

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