

Appendix A. Anthropogenic nitrogen deposition predicts local grassland primary production worldwide: Description of hierarchical modeling process.

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The hierarchical model predicting plot-level aboveground biomass was constructed by combining plot-level and site-level predictors as follows. The natural log of the live biomass harvested at peak standing crop (an estimate of aboveground annual productivity) was the response variable, and it was modeled for each plot i as

$$\ln(ANPP) \sim N(\mu_i, \sigma^2_{plot}) \quad (\text{Eq. A.1})$$

that is, a stochastic variable with expected value μ_i and a normally distributed plot error term. This plot expectation μ_i was derived from a site-level intercept (the modeled mean ANPP at a site), and plot-level response to edaphic conditions, which slopes could vary by site.

$$\mu_i = \alpha_{site} + \beta_{N_{site}} \text{soil}N_i + \beta_{P_{site}} \text{soil}P_i + \beta_{C_{site}} \text{soil}C_i + \beta_{pH_{site}} \text{soil}pH_i \quad (\text{Eq. A.2})$$

In matrix form,

$$\mu_i = B_{site} X_{plot} + \Sigma_B \quad (\text{Eq. A.3})$$

Where B is the vector of an intercept plus slopes depicting response to edaphic conditions, X is the vector of observed soil chemistry, and Σ_B contains the variance-covariance matrix. This matrix allows for group-level correlation among plot-level coefficients, in other words, for the expectation that variation in response to edaphic conditions is correlated within sites. We utilized a scaled inverse-Wishart model to satisfy the constraint that this matrix be positive definite (Gelman and Hill, 2007 pp377-380).

To include the site-level predictors, we modeled the full matrix \mathbf{B} of plot-level responses as deriving from a multivariate normal distribution with the same variance-covariance matrix:

$$\mathbf{B} \sim N(\widehat{\mathbf{B}}, \Sigma_{\mathbf{B}}) \quad (\text{Eq. A.4})$$

where $\widehat{\mathbf{B}}$ is the (site x plot predictor) matrix of the expected intercept and edaphic parameters, which are in turn predicted by site-level climate, elevation, and N deposition. The intercept, or mean productivity of the site, is thus:

$$\widehat{\mathbf{B}}_{[site,1]} = \hat{A} + \gamma_{Ndep} Ndep_{site} + \gamma_{MAT} MAT_{site} + \gamma_{MAP} MAP_{site} + \gamma_{PET} PET_{site} + \gamma_{elev} elev_{site} \quad (\text{Eq. A.5})$$

where \hat{A} represents the global mean productivity. In this case there is only one estimated (global) slope of response to each of N deposition, climatic factors and elevation. In matrix form,

$$\widehat{\mathbf{B}} = \mathbf{G}\mathbf{U} \quad (\text{Eq. A.6})$$

where \mathbf{G} is the matrix of influences of the global intercept and five site-level parameters on the site intercept and four plot-level parameters, and \mathbf{U} is the matrix of site climate, N deposition, and elevation values. From this notation it is clear the site-level predictors impact not only the site intercept or mean productivity, but the slopes of response of productivity to edaphic conditions, which makes sense biologically. These latter terms can be conceived of as

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“interactions” in the multiple regression sense—e.g. how does the plot-level response to soil pH change over the range of mean annual temperature or N deposition?

Finally, each of the elements in \mathbf{G} was modeled as a stochastic variable drawn from a normal distribution using an uninformative prior with mean zero and high variance:

$$g_{k,l} \sim N(0, \sigma^2_{\text{prior}}) \quad (\text{Eq. A.7})$$

with $\sigma^2_{\text{prior}} = 10^5$.

R code to implement the model can be found in supplementary materials.

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LITERATURE CITED

Gelman, A., and J. Hill. 2007. Data analysis using regression and multilevel/hierarchical models. Cambridge University Press, Cambridge ; New York.