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Appendix A: Derivation of a fjord-based, age structured model for cod (Gadus morhua L.) dynamics along the Norwegian Skagerrak coast.

Derivation of Eqns (1) through (4)

We now derive equations (2a)-(2c) from (1a)-(1c). First, we generalize (1a)-(1c) by replacing $Z_{f,t}$ by ($Z_{f,t} + \lambda x_{f,t}^{(0)}$) and incorporating the covariate species in (1b) to obtain

$$X_{s,t} = (Z_{f,t} + \lambda x^{(0)}{}_{f,t}) \, exp(\alpha_s + \alpha_t + \varepsilon_{s,t}) \qquad \text{(A1a)}$$

$$Y_{s,t} = X_{s,t-1} exp(-\beta x_{s,t-1} - \gamma y_{s,t-1} + \underline{\delta w}_{s,t-1}) \qquad \text{(A1b)}$$

$$Z_{f,t} = \Sigma m_t Y_{s,t-1} + \theta_t Z_{f,t-1}. \qquad \text{(A1c)}$$

In the log-scale, the preceding equations become

$$x_{s,t} = log(exp(z_{f,t}) + \lambda x^{(0)}{}_{f,t}) + \alpha_s + \alpha_t + \varepsilon_{s,t} \qquad \text{(A2a)}$$

$$y_{s,t} = (1-\beta) x_{s,t-1} - \gamma y_{s,t-1} + \underline{\delta w}_{s,t-1} \qquad \text{(A2b)}$$

$$z_{f,t} = log(\Sigma m \, exp(-F_{1,t-1} + y_{s,t-1}) + \theta \, exp(-F_{2,t-1} + z_{f,t-1})), \qquad \text{(A2c)}$$

where $x_{s,t} = log(X_{s,t})$, $y_{s,t} = log(Y_{s,t})$, and $z_{f,t} = log(z_{f,t})$; all summations sums over all sites s in a fjord f. Note that $m_t = m \, exp(-F_{1,t-1})$ *and* $\theta_t = \theta \, exp(-F_{2,t-1})$, where the *F*'s denote fishing mortality. Consider the case that $\lambda = 0$, $F_{1,t-1} \equiv 0$, $F_{2,t-1} \equiv 0$, and assume the simplified model is stationarity, with $\mu_{Y,s}$ and $\mu_{Z,f}$ being the stationary means of $Y_{s,t}$ and $Z_{f,t}$, respectively. Also, write $\mu_{y,s} = log(\mu_{Y,s})$ and $\mu_{z,f} = log(\mu_{Z,f})$. For the simplified model, taking expectation on both sides of Eq. (A1c) yields

$\mu_{Z,f}=\Sigma_s m\mu_{Y,s}+\theta\mu_{Z,f}$.

Hence, $(1-\theta)\mu_{Z,f}=\Sigma_s m\mu_{Y,s}$, which will be useful below. The basic idea for deriving (2a)-(2c) is to approximate the right side of (A2a)-(A2c) by a first order Taylor expansion around $\lambda=0, F_{1,t-1}=0, F_{2,t-1}=0, y_{s,t-1}=\mu_{y,s}, z_{f,t-1}=\mu_{z,f}$ and $z_{f,t}=\mu_{z,f}$. Equations (2a)-(2c) are obtained by noting that for (A2a), $\partial x_{s,t}/\partial z_{f,t}=1$ and $\partial x_{s,t}/\partial\lambda= x^{(0)}_{f,t}/\mu_{z,f}$, where all derivatives are evaluated at $\lambda=0, F_{1,t-1}=0, F_{2,t-1}=0, y_{s,t-1}=\mu_{y,s}, z_{f,t-1}=\mu_{z,f}$ and $z_{f,t}=\mu_{z,f}$. For equation (A2c),

$\partial z_{f,t}/\partial F_{1,t-1}=-(\Sigma m\mu_{Y,s})/(\Sigma_s m\mu_{Y,s}+\theta\mu_{Z,f}) =-(1-\theta)$, $\partial z_{f,t}/\partial F_{2,t-1}=-\theta\mu_{Z,f}/(\Sigma_s m\mu_{Y,s}+\theta\mu_{Z,f}) =-\theta$, $\partial z_{f,t}/\partial y_{s,t-1} =m\mu_{Y,s}/(\Sigma_s m\mu_{Y,s}+\theta\mu_{Z,f})$, and $\partial z_{f,t}/\partial z_{f,t-1}= \theta\mu_{Z,f}/(\Sigma_s m\mu_{Y,s}+\theta\mu_{Z,f}) =\theta$.

Define $c_s= m\mu_{Y,s}/[(\Sigma_s m\mu_{Y,s}+\theta\mu_{Z,f})(1-\theta)]$. Then it can be readily checked that $\Sigma_s c_s=1$. Hence, (A2a)-(A2c) approximately equal

$$x_{s,t} = z_{f,t} + \Delta_f x_{f,t}^{(0)} + \alpha_s + \alpha_t + \varepsilon_{s,t} \tag{2a}$$
$$y_{s,t} = (1-\beta)x_{s,t-1}-\gamma y_{s,t-1}+\underline{\delta w}_{s,t-1} \tag{2b}$$
$$z_{f,t} = d_f + (1-\theta)\Sigma c_s y_{s,t-1}+ \theta z_{f,t-1}- [(1-\theta)F_{1,t-1} + \theta F_{2,t-1}]; \tag{2c}$$

where $d_f = \mu_{z,f}-(1-\theta)\Sigma c_s\mu_{y,s} -\theta\mu_{z,f}$ is a constant, and $\Delta_f = \lambda/\mu_{Z,f}$. Consequently, we have

$$x_{f,t} = z_{f,t} + \Delta_f x^{(0)}_{f,t} + \alpha_f + \alpha_t + \varepsilon_{f,t} \tag{3a}$$
$$y_{f,t} = (1-\beta)x_{f,t-1}-\gamma y_{f,t-1}+\underline{\delta w}_{f,t-1} \tag{3b}$$
$$z_{f,t} = d_f + (1-\theta)y_{f,t-1}+ \theta z_{f,t-1}- [(1-\theta)F_{1,t-1} + \theta F_{2,t-1}], \tag{3c}$$

where the last equation [Eq. (3c)] is obtained by re-labeling some of the variables in Eqs. (2) so as to make all the model-variables fjord-specific (*i.e.*, $x_{f,t} = \Sigma c_s x_{s,t}$ and similarly we can define $y_{f,t}$).

Next, we outline the derivation of Eq. (4). Adding to the left side of (3c) the product of γ times the lag 1 of the left side of (3c) and doing likewise to the right side of (3c) eliminates the y's from the equation, because $y_{f,t}+\gamma y_{f,t-1}= (1-\beta)x_{f,t-1}+\underline{\delta w}_{f,t-1}$, owing to (3b). Specifically,

$$
\begin{aligned}
& z_{f,t}+\gamma z_{f,t-1} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\text{(A3)}\\
&\quad = (1+\gamma)d_f + (1-\theta)(y_{f,t-1}+\gamma y_{f,t-2})+ \theta(z_{f,t-1}+\gamma z_{f,t-2})- [(1-\theta)(F_{1,t-1} +\gamma F_{1,t-2})\\
&\quad + \theta(F_{2,t-1}+\gamma F_{2,t-2})]\\
&\quad =(1+\gamma)d_f + (1-\theta)[(1-\beta)x_{f,t-2}+\underline{\delta w}_{f,t-2}]+ \theta(z_{f,t-1}+\gamma z_{f,t-2})\\
&\quad -[(1-\theta)(F_{1,t-1}+\gamma F_{1,t-2})+ \theta(F_{2,t-1}+\gamma F_{2,t-2})].
\end{aligned}
$$

Equation (3a) implies that $z_{f,t} = x_{f,t} -(\Delta_f x^{(0)}_{f,t} + \alpha_f + \alpha_t + \varepsilon_{f,t})$. Upon substituting this expression into (A3), we obtain (4) after some algebra and noting that α_t is modeled as a linear combination of water temperature and the NAO:

$$
\begin{aligned}
x_{f,t} = {} & (\theta-\gamma)x_{f,t-1} + (\theta\gamma + (1-\theta)(1-\beta))x_{f,t-2} + (1-\theta)\underline{\delta w}_{f,t-2}\\
& +\Delta_f x^{(0)}_{f,t} + \Delta_f(\gamma-\theta)x^{(0)}_{f,t-1} - \Delta_f\theta\gamma x^{(0)}_{f,t-2}\\
& + const_f + \varepsilon_{f,t} + (\gamma-\theta)\varepsilon_{f,t-1} - \theta\gamma\varepsilon_{f,t-2}\\
& + \{-[(1-\theta)F_{1,t} + \theta F_{2,t}] - \gamma[(1-\theta)F_{1,t-1} + \theta F_{2,t-1}]\}\\
& + weather_t \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (4)
\end{aligned}
$$

where $weather_t = \eta_0 T_t + \eta_1 T_{t-1} + \eta_2 T_{t-2} + \kappa_0 nao_t + \kappa_1 nao_{t-1} + \kappa_2 nao_{t-2}$ (where further $\eta_0 = \eta$, $\eta_1 = (\gamma-\theta)\eta$ and $\eta_2 = -\gamma\theta\eta$, and $\kappa_0 = \kappa$, $\kappa_1 = (\gamma-\theta)\kappa$ and $\kappa_2 = -\gamma\theta\kappa$) and $const_f = (1+\gamma)d_f+ (1-\theta)(1+\gamma)\alpha_f$.

Appendix-Table 1a: A complete list of variables and their description.

Variables	
Symbol	**Description**
$\varepsilon_{s,t}$	stochastic effect (white noise) on cod reproduction at site *s* in year *t*
$\varepsilon_{f,t}$	(weighted) stochastic effect (white noise) on cod reproduction in fjord *f* in year *t*
I_t	dummy variable of the 1988 bloom; equals 1 for *t*=1988 and 0 otherwise
nao_t	Northern Atlantic Oscillation (NAO) index in year *t*
T_t	spring water temperature in year *t*
$w_{s,t}$	log abundance of covariate species at site *s* in year *t*
$w_{f,t}$	(weighted) log abundance of covariate species in fjord *f* in year *t*
$x_{f,t}^{(0)}$	amount (in million) of larvae released in fjord *f* in year *t*
$X_{s,t}$	0-group cod abundance at site *s* (within fjord *f*) in year *t*
$x_{s,t}$	log 0-group cod abundance at site s (within fjord *f*) in year *t*
$x_{f,t}$	(weighted) log 0-group cod abundance in fjord *f* and in year *t*
$Y_{s,t}$	1-group cod abundance at site *s* (within fjord *f*) in year *t*
$y_{s,t}$	log 1-group cod abundance at site *s* (within fjord *f*) in year *t*
$y_{f,t}$	(weighted) log 1-group cod abundance in fjord *f* and in year *t*
$Z_{f,t}$	abundance of adult and mature cod in fjord *f* in year *t*
$z_{f,t}$	log abundance of adult and mature cod in fjord *f* in year *t*

Appendix-Table 1b: A complete list of parameters and their description.

Parameters	
Symbol	**Description**
a_0, a_1, a_2	common direct (lag1, lag2) bloom effect on the (weighted) log 0-group abundance of any given fjord
A_f	direct bloom effect on the (weighted) log 0 -group abundance in fjord f
α_s	site effect in cod reproduction
α_t	year effect in cod reproduction
ar_1,ar_2	Lag 1 and lag 2 autoregressive coefficients in equation (5)
β	within-cohort intraspecific effect
B_f	lag-1 bloom effect on the (weighted) log 0 -group abundance in fjord f
c_s	relative 1-group cod abundance at site s within fjord f
B_f	lag-2 bloom effect on the (weighted) log 0 -group abundance in fjord f
d_t	intervention effect function of the 1988 bloom
δ	covariate species effects
Δ_f	fraction of the natural spawning population size that gives rise to 1 million larvae in fjord f
η	equals η_0
η_0, η_1, η_2	water temperature effects from current year, last year and two years ago.
$F_{1,t}$	fishing mortality rate of the 1-group cod
$F_{2,t}$	fishing mortality rate of the mature cod
γ	Between-cohort intraspecific effect
κ	equals κ_0
κ_0, κ_1, κ_2	NAO effects from current year, last year and two years ago.
λ	average number of mature cod needed to spawn 1 million larvae
ma_1,ma_2	lag 1 and lag 2 moving-average coefficients in equation (5)
$\mu_{Y,s}$	stationary mean 1-group abundance at site s under the simplified (1a-1b) with no trends nor covariates
$\mu_{y,s}$	equals log($\mu_{Y,s}$)
$\mu_{Z,s}$	stationary mean mature cod abundance in fjord f under the simplified (1a-1b) with no trends nor covariates
$\mu_{z,s}$	equals log($\mu_{Z,f}$)
m_t	survival rate from the 1-group cod to 2-group
ω	annual rate of change in fishing mortality of mature cod
π	common direct bloom effect on the 1-group cod
τ	common indirect bloom effect on the 1-group cod
θ	baseline survival rate of the mature cod (in year 1970)
θ_t	survival rate of adult and mature cod
ζ	common bloom effect on the 0-group cod

Appendix-Table 2 A: Parameter estimates obtained through the model fitting assuming, for reference, no effect of the algae bloom (see Chan *et al*. 2002b); $\Delta_f = \lambda/\mu_{Z,f}$. Bold p-values represent significance at the 5% level. Larvae releases are only performed in some fjords.

Parameter	**Estimate**	**SE**	**Ratio**	**p-value**	**Fjord name**
ma_1	-0.13	0.105	-1.23	0.226	
ar_2	0.45	0.090	5.02	**0.000**	
ma_2	-0.36	0.098	-3.68	**0.001**	
τ_1	-0.0073	0.0019	-3.90	**0.000**	
k_4	-0.12	0.048	-2.53	**0.015**	
η_0	-0.073	0.054	-1.37	0.177	
η_1	0.098	0.056	1.75	0.086	
η_2	0.13	0.058	2.21	**0.032**	
κ_0	0.043	0.031	1.41	0.164	
κ_1	-0.068	0.033	-2.02	**0.049**	
κ_2	-0.055	0.037	-1.49	0.143	
Δ_1	0.13	0.054	2.33	**0.024**	Torvefjord
Δ_2	-0.041	0.030	-1.39	0.171	Topdalsfjord
Δ_3	0.036	0.017	2.13	**0.038**	Høvåg
Δ_4	0.045	0.014	3.30	**0.002**	Bufjord
Δ_5	-0.0033	0.0097	-0.34	0.735	Flødevigen
Δ_7	-0.0047	0.0086	-0.54	0.590	Sandnesfjord
Δ_8	0.021	0.018	1.21	0.234	Søndeledfjord
Δ_{11}	0.016	0.27	0.57	0.569	Kilsfjord
Δ_{16}	-0.011	0.010	-1.11	0.272	Nøtterø
Δ_{17}	0.018	0.0061	2.99	**0.004**	Holmestrand area
Δ_{20}	-0.0086	0.0046	-1.88	0.067	Hvaler

Appendix-Table 2 B: Parameter estimates of the ecological parameters obtained from the equating the coefficients in Eqs. (4) and (5). Bold p-values represent significance at the 5% level.

Term	**Estimate**	**SE**	**Ratio**	**p-value**
θ	0.67	0.071	9.49	Term
γ	0.54	0.11	4.77	Term
β	0.73	0.094	7.80	Term
Pollack old	-0.37	0.16	-2.35	Term