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Appendix B. Multilevel regression example.

Multilevel, or hierarchical, regression models allow for the analysis of group-structured datasets. This approach offers several concrete advantages for investigating how effects of interest vary by group, incorporating the structure of non-independent data in an appropriate way, and estimating regression parameters and uncertainty measures. As an example, the regression model that includes system and response mechanism as grouping factors can be written as follows (Gelman and Hill 2007):

$$y_i = \left(\beta + \gamma 1_{j[i]} + \gamma 2_{k[i]}\right) x_i + \varepsilon_i ,$$

or equivalently,

$$y_i \sim N\left(\left(\beta + \gamma 1_{j[i]} + \gamma 2_{k[i]}\right) x_i,\ \sigma_y\right),$$

with $\gamma 1_j \sim N\left(0, \sigma^2{}_{\gamma 1}\right)$, $\gamma 2_j \sim N\left(0, \sigma^2{}_{\gamma 2}\right)$, and $\varepsilon_i \sim N\left(0, \sigma^2{}_y\right)$.

Here y_i is ln(C_p/C_b) for observation *i*, x_i is ln(R_p/R_b) for observation *i*, β is the mean slope, $\gamma 1_{j[i]}$ is the deviate from the mean slope for system *j* of which observation *i* is a member, $\gamma 2_{k[i]}$ is the deviate from the mean slope for response mechanism *k* of which observation *i* is a member, ε_i is the residual error for observation *i*, and $\sigma^2{}_y$ is the variance for the data-level error. Additionally, $\sigma^2{}_{\gamma 1}$ is variance of the slope deviates for the system factor, and $\sigma^2{}_{\gamma 2}$ is variance of the slope deviates for the response mechanism factor. Conceptually, this model may be written as:

$\ln(C_p/C_b)$ = (mean slope + deviation in slope due to system + deviation in slope due to response mechanism)* $\ln(R_p/R_b)$

Fitting this model simultaneously estimates the mean relationship between $\ln(C_p/C_b)$ and $\ln(R_p/R_b)$, the effect of system on this relationship for all 36 systems, and the effect of response mechanism on this relationship for our three response mechanisms. It will also estimate variance due to system, variance due to response mechanism, and residual variance. With this approach the effects of system and response mechanism, $\gamma 1_j$ and $\gamma 2_j$ respectively, are modeled as random effects with respective variances $\sigma^2_{\gamma 1}$ and $\sigma^2_{\gamma 2}$. By treating these effects as random, we are assuming that the effects are drawn from a common distribution with a variance estimated from the data. We consider this assumption to be appropriate because we are treating consumer responses to resource pulses as a widespread phenomenon whose details may vary due to the characteristics of individual systems. As the deviation of a group effect from the mean effect is weighted by the sample size of that group (Gelman and Hill 2007), we can make use of unbalanced data to conservatively estimate the effects of system and response mechanism.

Literature Cited

Gelman, A., and J. Hill. 2007. Data analysis using regression and multilevel/hierarchical models. Cambridge University Press, New York, New York, USA.