APPENDIX

A Notes on Loosmore and Ford's test statistic

A.1 Sums and integrals

Loosmore and Ford (2006, eq. (3)) propose the test statistic

$$U_{\text{obs}} = \sum_{r_k=r_{\min}}^{r_{\max}} (H_{\text{obs}}(r_k) - H_{\text{theo}}(r_k))^2 \delta r_k \tag{A.1}$$

where $r_1, \ldots, r_N$ is a sequence of distance values at which $H_{\text{obs}}(r)$ and $H_{\text{theo}}(r)$ have been computed, and $\delta r_k = r_{k+1} - r_k$ is the distance increment. For the simulated point patterns the corresponding values are

$$U_i = \sum_{r_k=r_{\min}}^{r_{\max}} (H_i(r_k) - H_{\text{theo}}(r_k))^2 \delta r_k. \tag{A.2}$$

For a f nely spaced sequence of distance values, the sum in (A.1) is a close approximation to the *integral*

$$U_{\text{obs}} = \int_{r_{\min}}^{r_{\max}} (H_{\text{obs}}(r) - H_{\text{theo}}(r))^2 \, \mathrm{d}r \tag{A.3}$$

and similarly

$$U_i = \int_{r_{\min}}^{r_{\max}} (H_i(r) - H_{\text{theo}}(r))^2 \, \mathrm{d}r. \tag{A.4}$$

A.2 Sample means

When the theoretical value $H_{\text{theo}}(r)$ is not known, Loosmore and Ford (2006) propose replacing $H_{\text{theo}}(r)$ in (A.1) by the average of the simulated values

$$\bar{H}(r) = \frac{1}{m}\sum_{i=1}^{m} H_i(r). \tag{A.5}$$

Thus for a f ne sequence of r values the test statistic is

$$U_{\text{obs}} = \int_{r_{\min}}^{r_{\max}} (H_{\text{obs}}(r) - \bar{H}(r))^2 \, \mathrm{d}r. \tag{A.6}$$

To preserve symmetry in the treatment of the observed and simulated data, let us treat the observed point pattern as pattern 0, and write $H_{\text{obs}}(r)$ as $H_0(r)$. Then $\bar{H}(r)$ is the average of all the functions $H_i(r)$ *except* $H_0(r)$. Symmetry requires that $H_{\text{theo}}(r)$ in (A.2) should be replaced by

$$H_{-i}(r) = \frac{1}{m}\sum_{j \neq i} H_j(r) \tag{A.7}$$

where the index j may include 0. Thus

$$U_i = \int_{r_{\min}}^{r_{\max}} (H_i(r) - H_{-i}(r))^2 \, \mathrm{d}r. \tag{A.8}$$

The computation can be simplif ed by noticing that

$$H_{-i}(r) = \frac{1}{m}\left(\sum_{j=0}^{m} H_j(r) - H_i(r)\right) = \frac{m+1}{m}\overline{\overline{H}}(r) - \frac{1}{m}H_i(r) \tag{A.9}$$

where

$$\overline{\overline{H}}(r) = \frac{1}{m+1} \sum_{j=0}^{m} H_j(r) \tag{A.10}$$

is the average of all the observed and simulated functions. This gives

$$H_i(r) - H_{-i}(r) = H_i(r) - \frac{m+1}{m}\overline{\overline{H}}(r) + \frac{1}{m} H_i(r) = \frac{m+1}{m}\left(H_i(r) - \overline{\overline{H}}(r)\right). \tag{A.11}$$

Thus, the statistic of Loosmore and Ford (2006) is equivalent to

$$U_{\text{obs}} = a \int_{r_{\min}}^{r_{\max}} (H_{\text{obs}}(r) - \overline{\overline{H}}(r))^2 \, \mathrm{d}r \tag{A.12}$$

$$U_i = a \int_{r_{\min}}^{r_{\max}} (H_i(r) - \overline{\overline{H}}(r))^2 \, \mathrm{d}r \tag{A.13}$$

where $a = ((m+1)/m)^2$ and

$$\overline{\overline{H}}(r) = \frac{m}{m+1}\bar{H}(r) + \frac{1}{m+1} H_{\text{obs}}(r) \tag{A.14}$$

is the average of all the simulated and observed values. This reduces the computational load because it does not require calculation of $H_{-i}(r)$ for each i.

Literature cited

Loosmore, N. B., and E. D. Ford. 2006. Statistical inference using the G or K point pattern spatial statistics. *Ecology* 87:1925–1931.