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Appendix B. Autocorrelation for u'_t, an estimator of u_t by the moving-average method. A pdf file of this appendix is also available.

Figure A2 (see Appendix A) shows the sample autocorrelation functions, $r_{uu}(l)$ and $r_{u'u'}(l)$, for the density-independent influence u_t and its estimator u'_t (Fig. 6b). The $r_{uu}(l)$, as samples of $r_{uu}(l)$ ° 0, l > 0, are insignificantly different from zero at all lags shown, whereas the $r_{u'u'}(l)$ exhibit a significant negative spike at lag 2, which can be explained as follows.

Consider that the series $\{R_t\}$ is made up of two components: a low-frequency cycle (g_t, say) due to the autocorrelated density-dependent process and a rapid fluctuation (about the cycle) due to the density-independent influence u_t with no autocorrelation. Thus, we may write

$$R_t = g_t + u_t. \tag{A.2}$$

After the moving-average transformation on both sides of (A2), we have

$$R'_t = g'_t + u''_t \tag{A.3}$$

in which the single and double primes indicate the moving-average transformations of the respective letters in Eq. A.2. Recalling the definition $u'_t = R_t - R'_t$ in the main text, the difference (A.2) - (A.3) yields

$$g_t - g'_t + u_t - u''_t = u'_t.$$

Now that, by assumption, g_t is void of the rapidly fluctuating component, the difference

$|g_t - g'_t|$ would be small compared with $|u_t - u''_t$ (= z_t, say)| such that u'_t would be approximately equal to z_t. Hence, letting k (an odd positive integer) be the order of the moving-average transformation, the autocorrelation function $r_{u'u'}(l)$ would be approximated by $r_{zz}(l)$ whose theoretical function is readily shown to be

$$\begin{aligned} r_{zz}(l) &= -(k+l)/k(k-1), && 0 < l \leq (k-1)/2 \\ &= (k-l)/k(k-1), && (k-1)/2 < l < k \\ &= 0, && k < l. \end{aligned} \tag{A.4}$$

For $k = 5$, $r_{zz}(1) = -0.3$, $r_{zz}(2) = -0.35$, $r_{zz}(3) = 0.1$, $r_{zz}(4) = 0.05$, and $r_{zz}(l \geq 5) = 0$, as in the $r_{u'u'}(l)$ based on the length 50, although for length 4000 only the $r_{u'u'}(2)$ is distinct. The tendency for $r_{u'u'}(2)$ to be distinctly negative shows up in the cross-correlation function $r_{u'v'}(l)$ (Fig. 7, middle column) that tends to exhibit significant spikes at lags other than 0.

The results (A.4) show that $r_{zz}(l)$ tends to zero for all l as the order k increases, apparently suggesting that the higher the order k, the better $r_{u'v'}(l)$ approximates $r_{uv}(l)$. This is true when g_t is linear, or flat, across all t. Otherwise, g'_t would over-smooth g_t sooner or later as k increases, resulting in the difference ($g_t - g'_t$) to increase in variance. Conversely, too low an order results in an under-smoothing that also results in an increase in variance. Thus, there must be an optimal order at which the variance of the difference (g_t - g'_t) is minimized. Delving further into this subject is beyond the present scope. However, a graphical inspection (cf. Fig. 6a) would probably be adequate in practice.

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